Below is Midterm II of Spring 2003. Do Questions 1, 2(b), 3 and 4 (total 40 points). Question 2(a) is a bonus question (4 bonus points) for this homework. It is about the normal distribution. You can read the relevant section from the "free" textbook, Chapter 5, pages 212 to 215.

QUESTION 1 [Total 10]

(a) [3+3] Thomas' girlfriend, Connie, is a fish lover. For Connie's birthday, Thomas plans to prepare a "fish feast" for her, which would require 3 fresh fish. To get the fresh fish, Thomas is going fishing this Saturday, and, from past experience, the number of fish that Thomas can catch in one day follows a Poisson distribution with mean 1.7.
(i) What is the chance that Thomas will catch 2 fish (and hence poor Thomas would need to buy a third fish from the fish market)?
(ii) What is the chance that Thomas will catch 3 or more fish?

(b) [4] The following Splus program concerns an experiment with two fair dice. What is this program trying to do?

```
program1 <- function(N=10000)
{
  ans <- 0
  for (i in 1:N) {
    die1 <- sample(1:6, 1, replace=T)
    die2 <- sample(1:6, 1, replace=T)
    x <- max(die1,die2)-min(die1,die2)
    ans <- ans + x
  }
  return(ans/N)
}
```

QUESTION 2 [Total 10]

(a) [4] Thomas is supposed to meet his girlfriend Celeste at 7:30pm. Suppose the time requires Thomas to drive from his office to Celeste's place follows a normal distribution with mean 12 minutes and standard deviation 2 minutes. If Thomas leaves his office at 7:15pm, what is the chance that he will be late and badly tortured by Celeste?

(b) [6] There are three coins in an urn. Two of the coins have head on both sides while the remaining coin has head on one side and tail on the other side. A coin is randomly selected and you are allowed to look at one of its faces. Suppose you see a head. What is the probability that the other face of the selected coin is a tail?

QUESTION 3 [Total 10]

Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} k(6x+4) & \text{if } 0 \le x \le c, \\ 0 & \text{otherwise,} \end{cases}$$

where both k and c are constants greater than zero. It is known that $E(X) = \frac{4}{7}$.
(a) [3] Show that $k = \frac{1}{7}$ and $c = 1$.
(b) [4] Find $V(X)$.
(c) [3] Suppose $Y = 2X^2 + 3X - 5$. Find $E(Y)$.

QUESTION 4 [Total 14]

I have three fair dice. Die A has four faces marked "1" and two faces marked "2"; Die B has three faces marked "1" and three faces marked "2"; and Die C has two faces marked "1", three faces marked "2" and the remaining face marked "3". The following experiment is performed: I first randomly choose one of these three dice, and then I throw the chosen die repeatedly. Let X be the number obtained from the first throw.

(a) [3] Find $P(X = 1)$.
(b) [3] If $X = 1$, what is the probability that Die A is the chosen die?
(c) [4] What is $E(X)$?
(d) [4] If $X = 1$, find the probability of getting a "1" on the second throw.