Max-stable processes and annual maximum snow depth

Juliette Blanchet

blanchet@slf.ch

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Outline

Motivation

Max-stable process - Schlather model

Annual maximum snow depth in Switzerland

Conclusion and outlook

SLF research

Swiss Federal Institute for
Snow and Avalanche
Research, Davos
`www.slf.ch`

- Avalanche Warning and Prevention
- Snow Physics, Permafrost and Climatology
- Avalanches, Debris Flows and Rockfall
- Mountain Hydrology and Torrents
- Ecosystem Boundaries

Extreme snowfall danger

Japan, January 2006

Hautes-Alpes, January 2004

- Avalanche warning and prevention
- Settlement and infranstructure management
- Floods and landslides caused by snowmelt

Our goal

- Try to understand how extreme snow depth is distributed over Switzerland
- Get return levels maps
- Simulate extreme snow depth
- ...

A spatial model is required!

Max-stable process - Schlather model

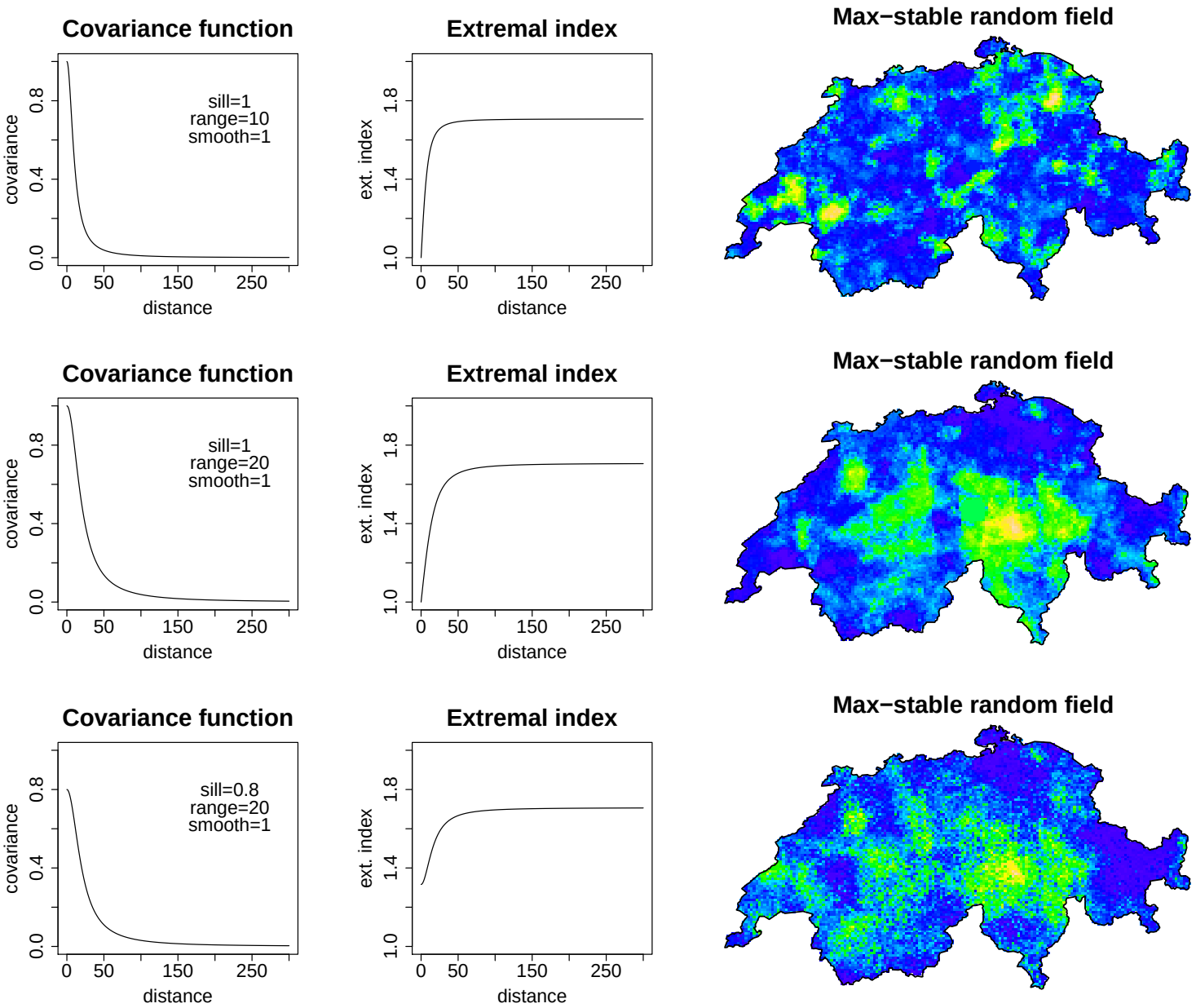
- $Z(x)$ process of block maxima (ex: maximum annual snow depth) at location x, assumed to be unit Fréchet
- Schlather model for the bivariate CDF in locations x_1 and x_2:

$$P\left[Z(x_1) \leq z_1, Z(x_2) \leq z_2\right]$$
$$= \exp\left[-\frac{1}{2}\left(\frac{1}{z_1}+\frac{1}{z_2}\right)\left(1+\sqrt{1-2(\rho(h)+1)\frac{z_1 z_2}{(z_1+z_2)^2}}\right)\right]$$

Covariance function $\rho(h)$: how decreases the dependence between two locations when the distance increases.
$\rho(h) \in [0,1]$, $\rho(h) \searrow 0$ when $h \nearrow$ (ex: Cauchy, ...)

Extremal coefficient $\theta(h)$:
$\theta(h) = 1 + \sqrt{\frac{1-\rho(h)}{2}} \in [1, 1+\sqrt{0.5}] \simeq [1, 1.7]$
$\Rightarrow$ Complete independence ($\theta = 2$) is **never achieved**

Simulation of max-stable processes



Sensibility of Fréchet transform

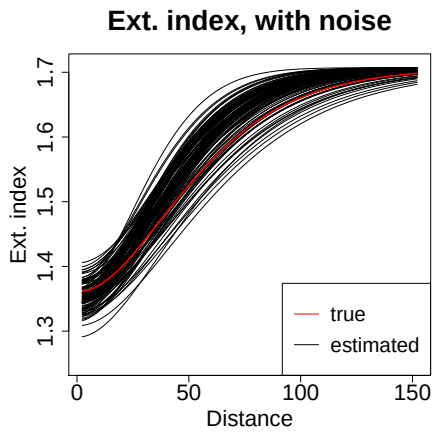
- If $Y(x) \sim GEV(\mu(x), \sigma(x), \xi(x))$ then $Z(x) = \left[1 + \xi(x)\frac{Y(x)-\mu(x)}{\sigma(x)}\right]_+^{\frac{1}{\xi(x)}} \sim$ Fréchet(1)
- Schlather model gives the bivariate distribution of $Z(x_1)$ and $Z(x_2)$
- Approximation of the full likelihood by the pairwise likelihood (*SpatialExtremes* package)

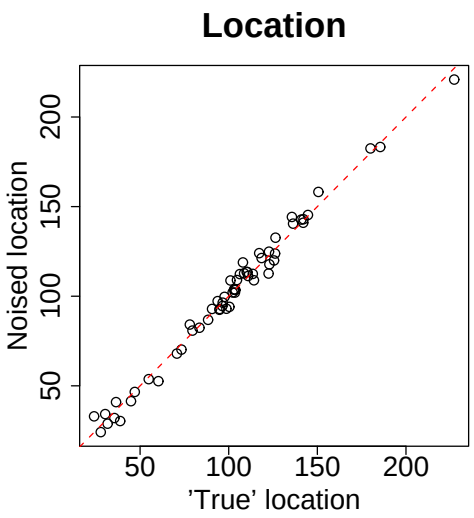
The covariance estimation is sensible to errors on the GEV parameters.

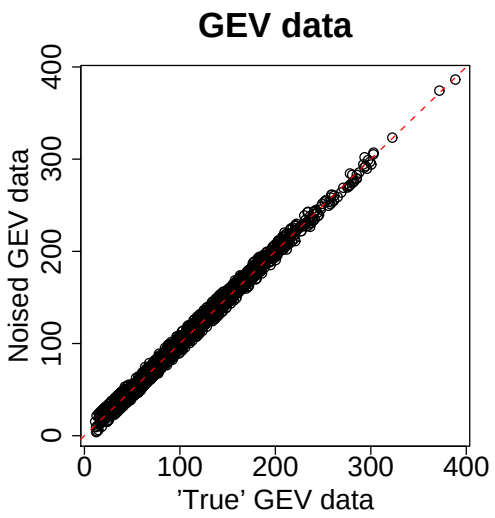
A (very) good GEV model is needed!

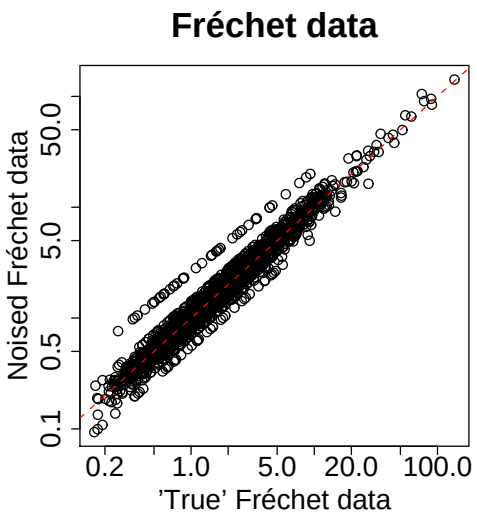
Simulation: influence of noise in covariance estimation

$$\mu(x) = \mu^0(x) + N(0, \sigma = 5)$$



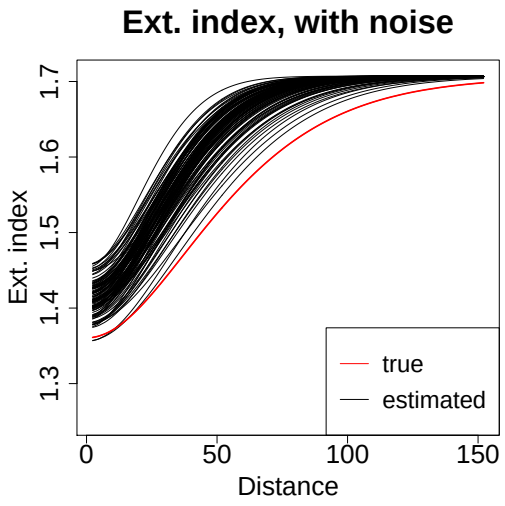


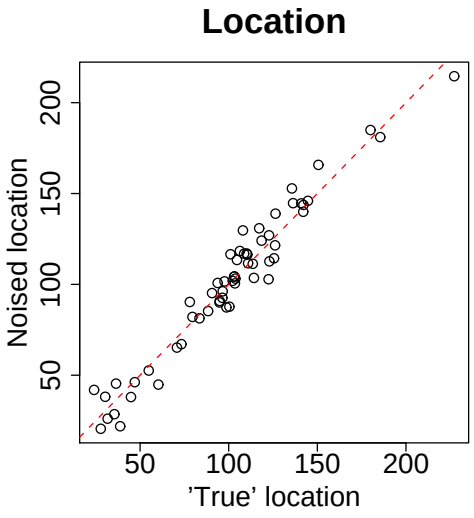


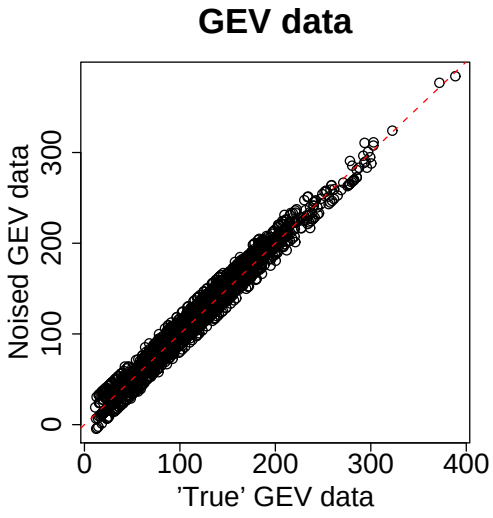


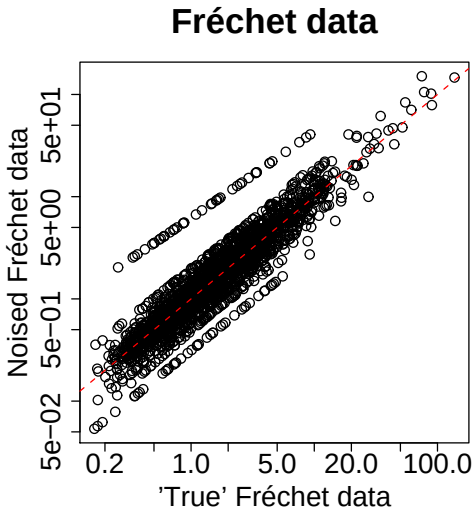
Simulation: influence of noise in covariance estimation

$$\mu(x) = \mu^0(x) + N(0, \sigma = 10)$$



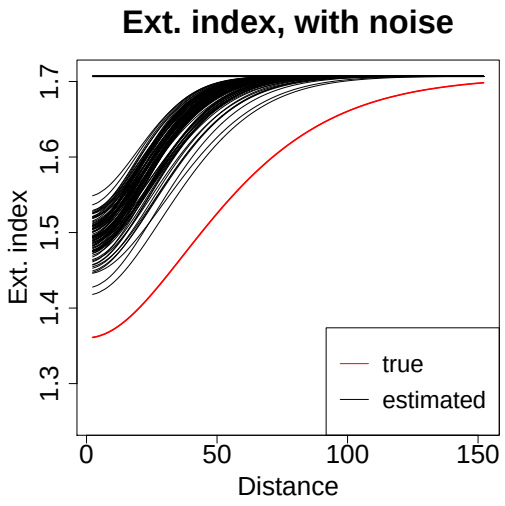


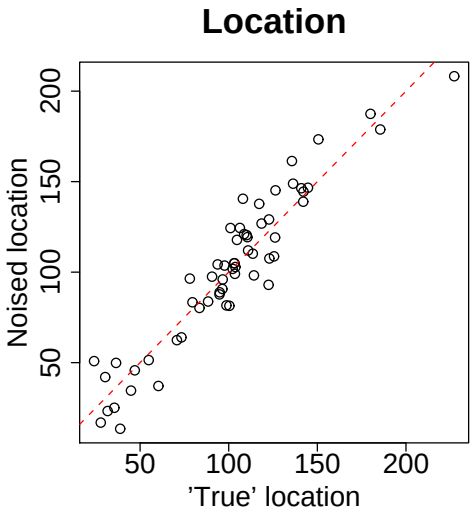


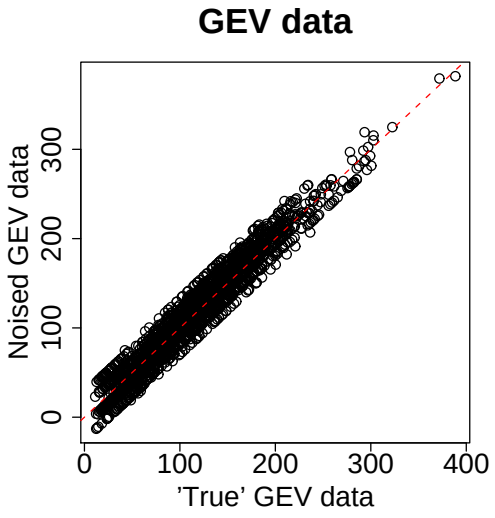


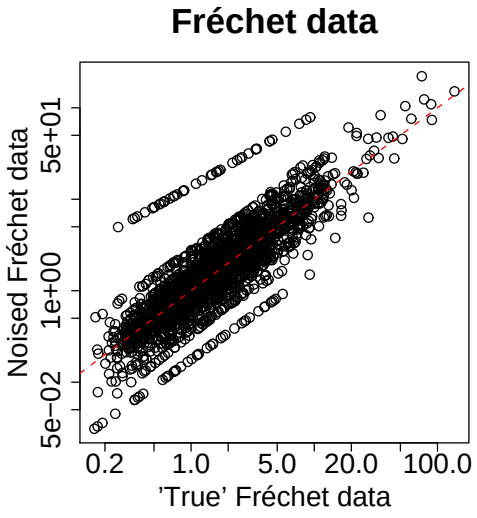
Simulation: influence of noise in covariance estimation

$$\mu(x) = \mu^0(x) + N(0, \sigma = 15)$$



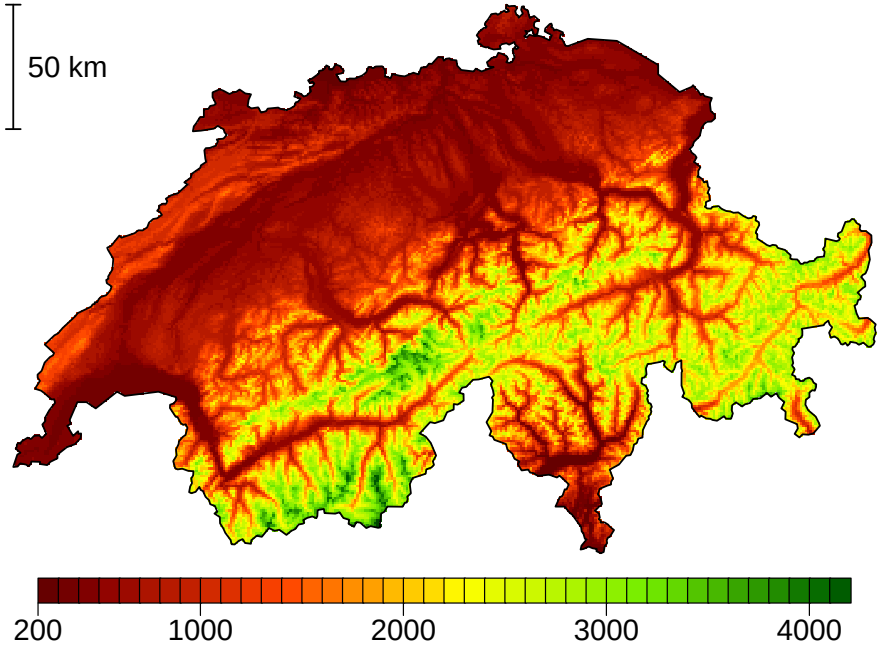


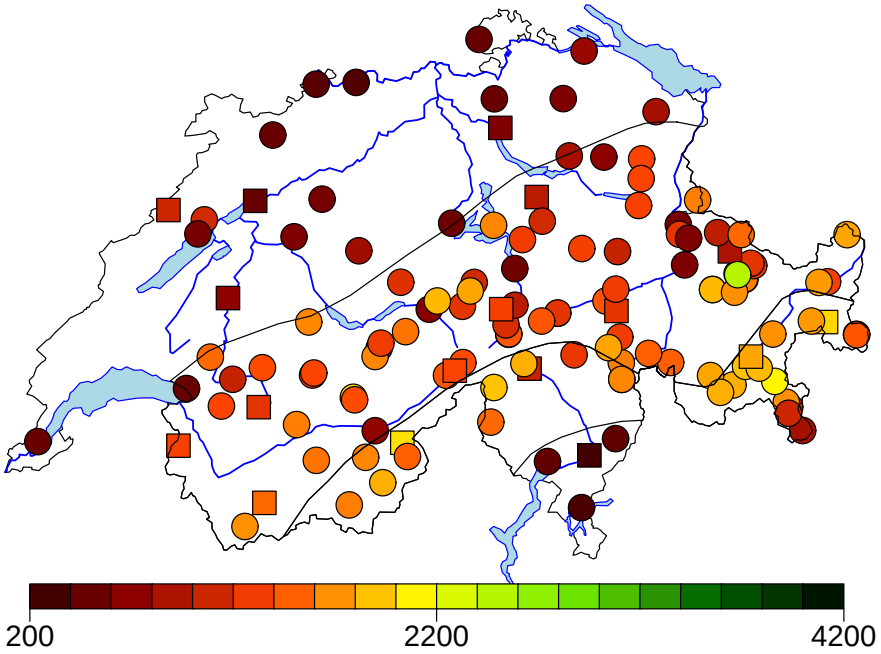




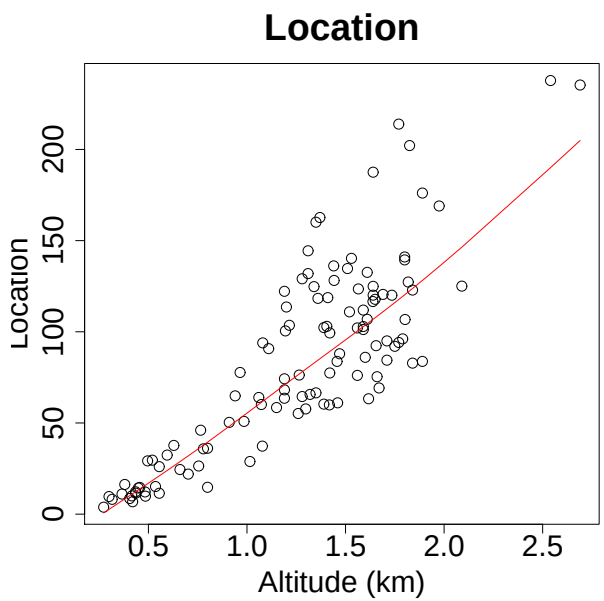
Data

- 130 stations in the Alps
- 43 winters (1966-2008)
- 4 regions: Plateau, North slope/South slope, Tessin
- 17 stations are removed from the analysis for validation

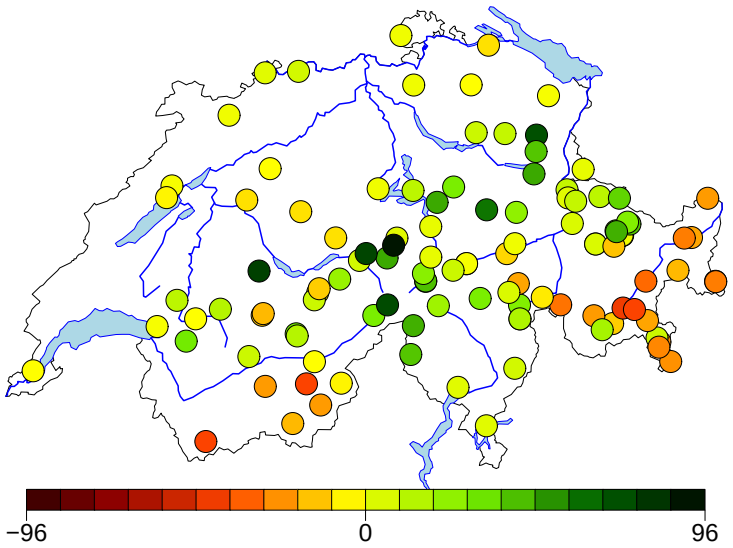




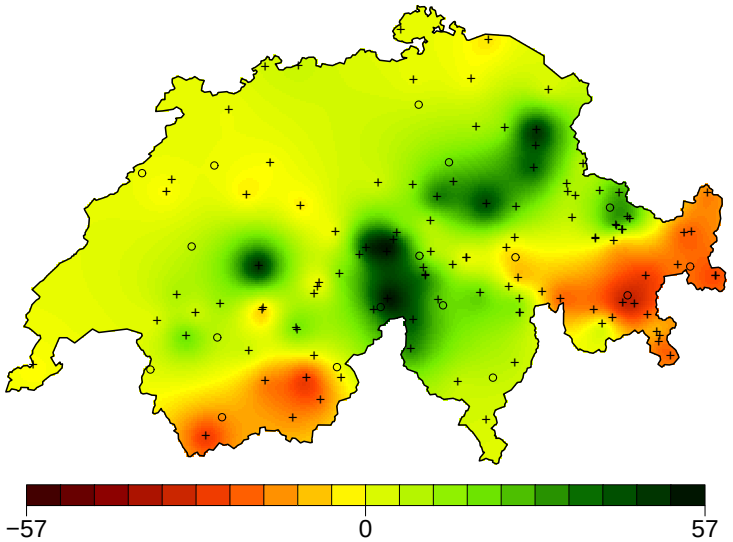
GEV model: Location



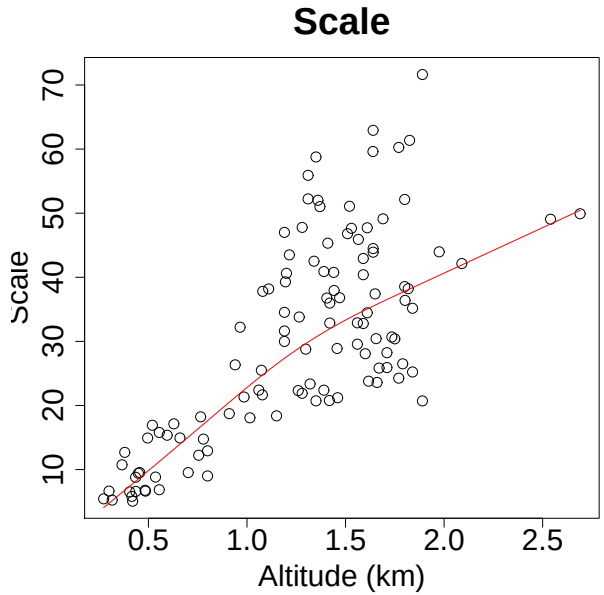
Residuals



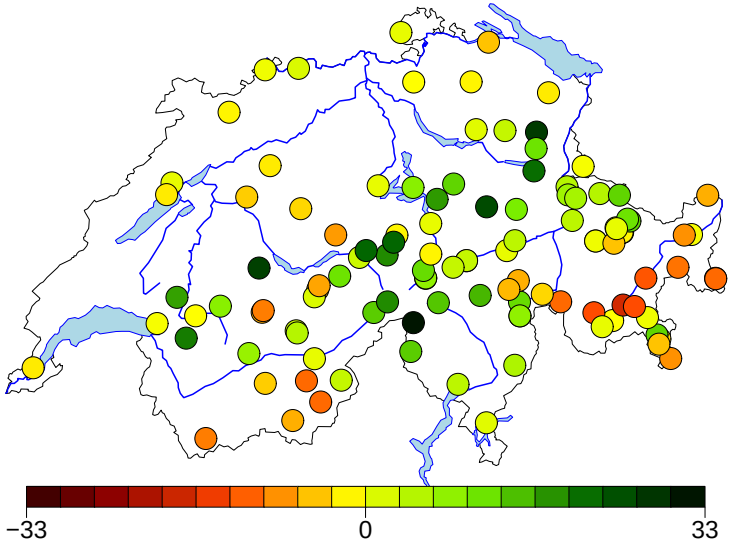
Kriging residuals



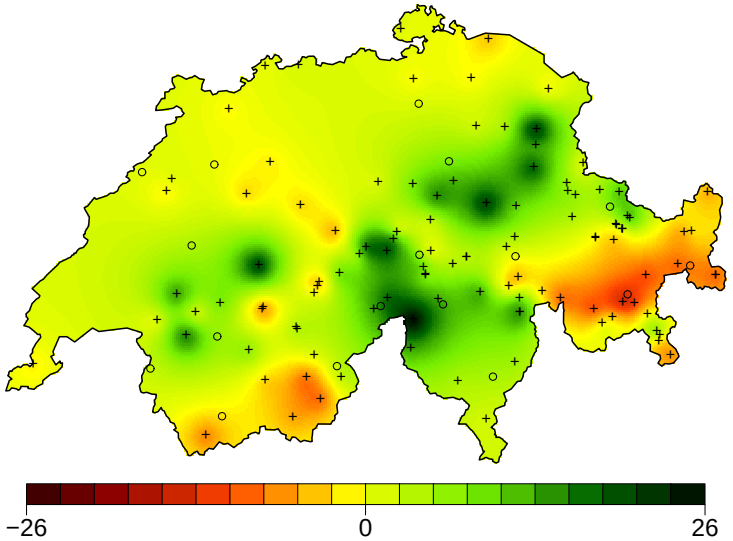
GEV model: Scale



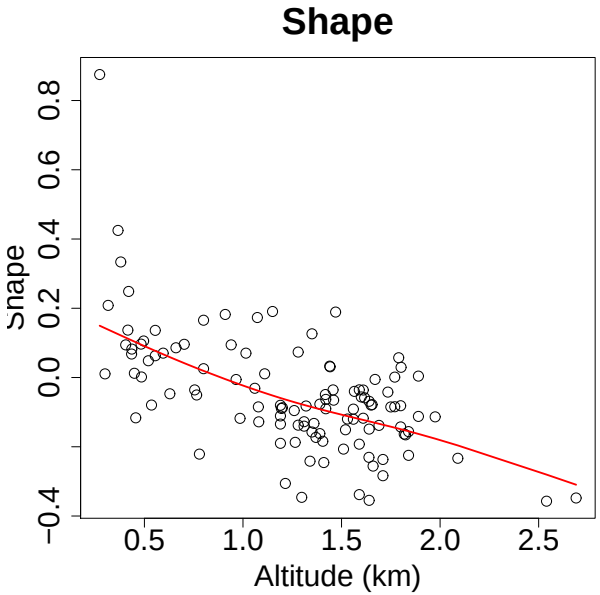
Residuals



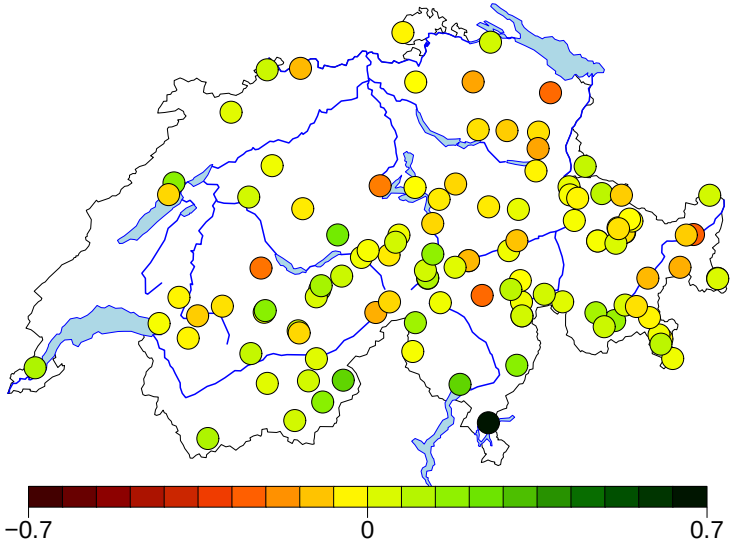
Kriging residuals



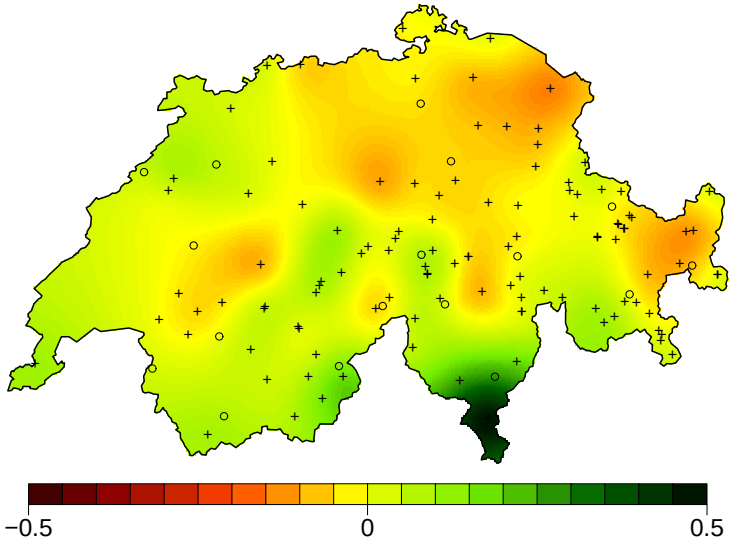
GEV model: Shape



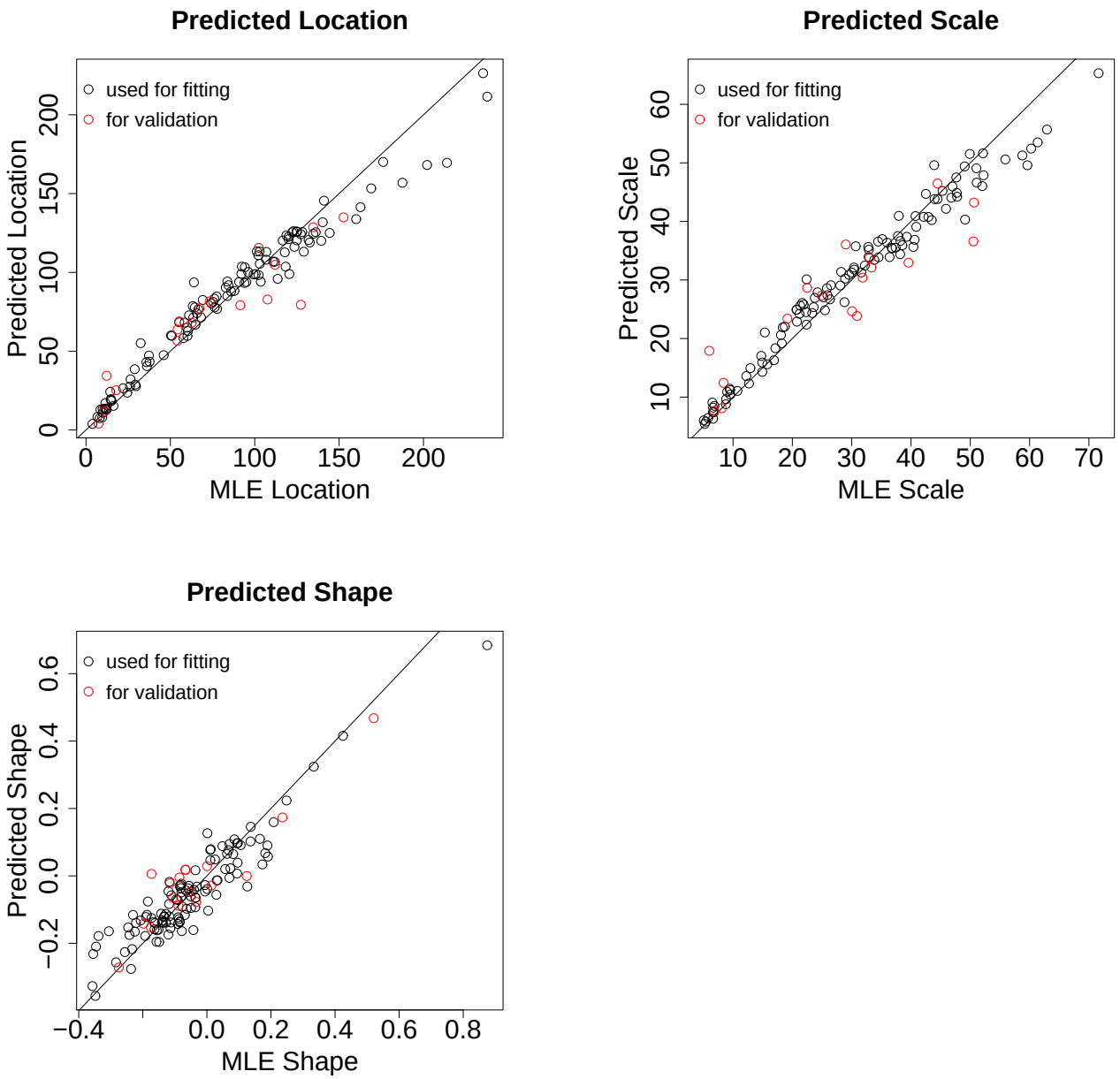
Residuals



Kriging residuals



GEV model: Validation



Schlather model estimation

$$P\left[Z(x_1) \leq z_1, Z(x_2) \leq z_2\right]$$
$$= \exp\left[-\frac{1}{2}\left(\frac{1}{z_1}+\frac{1}{z_2}\right)\left(1+\sqrt{1-2(\rho(h)+1)\frac{z_1 z_2}{(z_1+z_2)^2}}\right)\right]$$

h: distance between x_1 and x_2

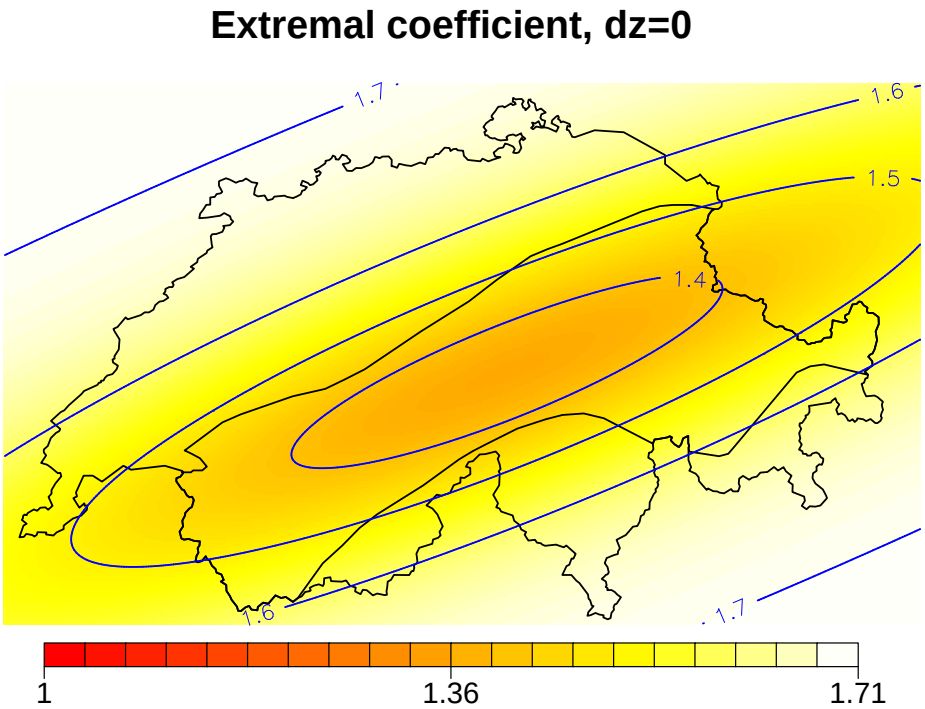
"Climatic" space: $h = \sqrt{\Delta x^T A \Delta x}$ where

$$A = \begin{pmatrix} \cos\phi & r\sin\phi & 0 \\ -r\sin\phi & \cos\phi & 0 \\ 0 & 0 & s \end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix} \text{longitude} \\ \text{latitude} \\ \text{altitude} \end{pmatrix}$$

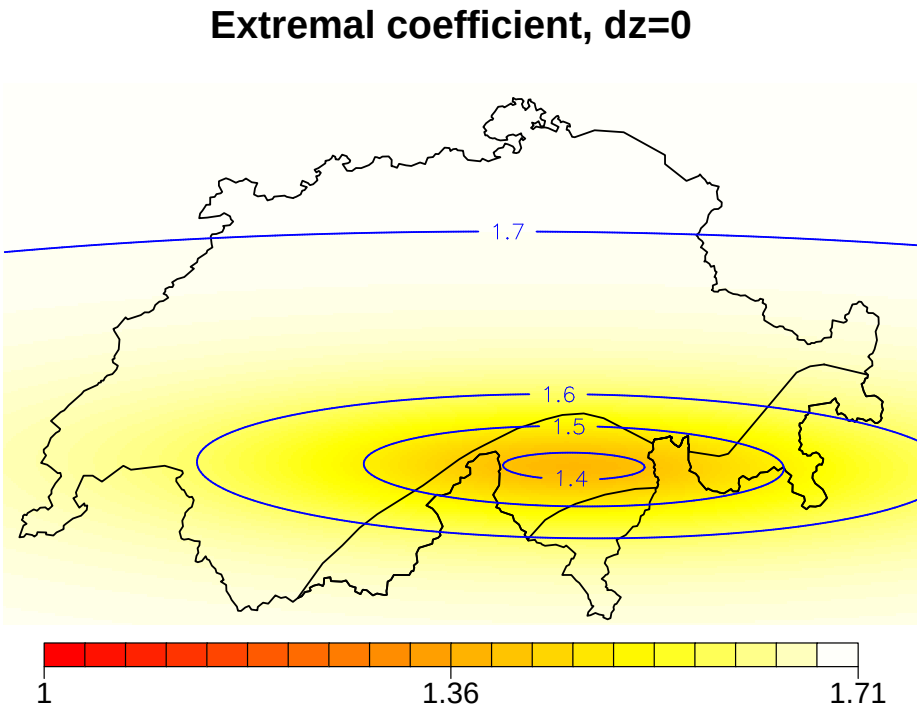
$\rightarrow$ extremal index $\theta = c$ is an ellisoid

Extremal index in the Alps

North Slope of the Alps:
Pract. range = 750km in "x"
= 150km in "y" = 2000m in z



South Slope of the Alps
Pract. range = 620km in "x"
= 120km in "y" = 2800m in z



+ Simulations, conditional maps, ...

Conclusion

- Max-stable model for extreme snow depth
- Requires a good GEV model
- Illustration in Switzerland

Outlook:

- Introduce topography in covariance function
- Covariate for the GEV????