Dynamic network sampling

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  often has network structure
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  often has network structure
  and moves or changes over time
- Designs for selecting a **sample** units
  use spatial and network relationships and progress dynamically

Population and sample processes

Population: A stochastic process $\{Y_t\}$.

Sample: A stochastic process $\{S_t\}$.

Time t such as day, with $t = 0, 1, 2, \ldots$.

Values Y_t of units such as locations, states, and relationships between units at time t.

Sample S_t the set of units in the sample at time t.

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- clustering, mixing, migration—point process
- movements within and among groups—small random displacements, MCMC selections, autoregressive processes
- insertions and deletions of objects—birth and death process, immigration and emigration.

Dynamic network model

- builds on spatial temporal point process
- link probabilities dependent on distance between nodes, node characteristics, and current degree or target degree distribution
- renewal process for link formation, persistence, and dissolution

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- For the dynamic situation we use a sampling process that includes
  - an **acquisition process** by which units are added to the sample
  - an **attrition process** by which units are removed from the sample

Equilibrium distributions

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- Sample size tends to increase when acquisition rate exceeds attrition rate
- and decrease when attrition rate exceeds attrition rate

Uses of sampling designs

- **Inference** about population characteristics
- **Experiments** on sample units
- **Interventions** on sample units

Intervention strategy

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- Objective is to change the population, not just sample units, in a desired way.
- One strategy interacts with another

Effect of an intervention

- An **intervention strategy** consists of a sampling design for finding units in the population on which to make interventions, and a plan for the types of interventions to be made, which may depend on sample unit characteristics.
- A simple way to view the **effect** of a strategy is the difference in the resulting equilibrium distribution, compared with the equilibrium distribution without the strategy, or with a different strategy.

Natural sampling strategies

- virus selects a sample of people with a link-tracing design
- insects select a sample of plants with a temporal spatial distance design.

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- units are removed from sample through removal probability or deletion from population.

Random walk in static network

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- Connected components of networks form closed classes.

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- links between nodes change over time
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- links between nodes change over time
- component structure changes
- a random walk temporarily stuck in one component can eventually reach nodes of other components
- deletions and insertions of nodes can interrupt the walk, requiring reseeding

Random set designs

- current sample S_t is a set of nodes
- acquisition of nodes includes tracing links out from sample
- attrition through removal of nodes from sample or deletion from population

Simple random set design

- with-replacement selection
- independent link tracing
- independent removals

Design options

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- with or without replacent
- links followed from active sample units only
- replacement, activeness values between 0 and 1

Desired features

- trace rapidly at first; reseeding.
- have a target sample size distribution.
- find units with high degree or "interesting" values.

Epidemic example

- HIV virus spreads with dynamic network sampling design
- seek and treat designs for interventions to reduce incidence.
- combination of interventions and counter-responses leads to new equilibrium distribution.

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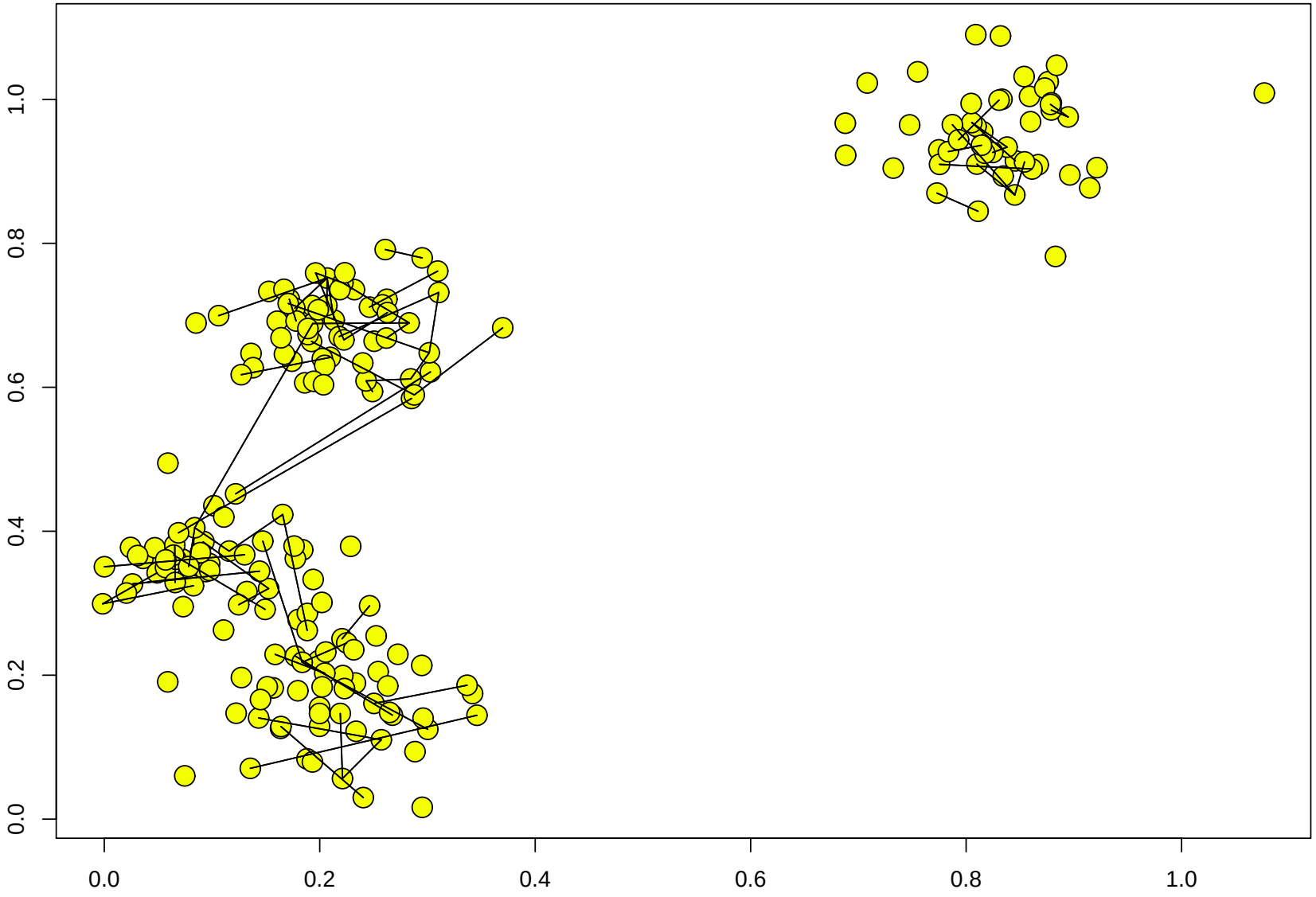
- sample volume = number of nodes
- sample surface = number of links out
- may be weighted by tracing probabilities
- surface to volume ratio tends to decrease as sample size increases

Also affecting equilibrium level

- new node entering sample (HIV) tends to have higher degree than average for population
- and higher proportion of links-out than average for sample
- especially in early stages of epidemic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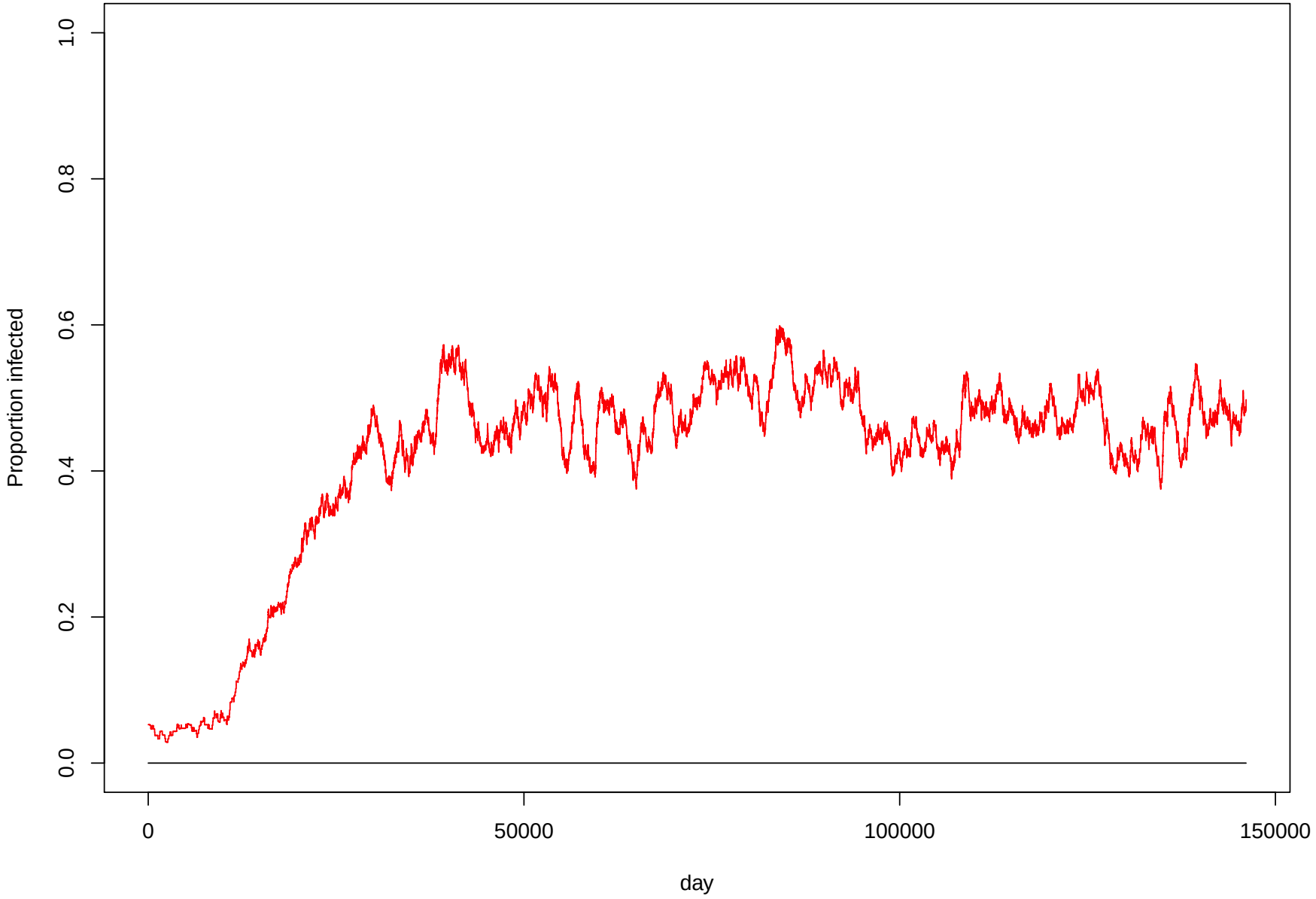
Proportion infected
0.0
0.2
0.4
0.6
0.8
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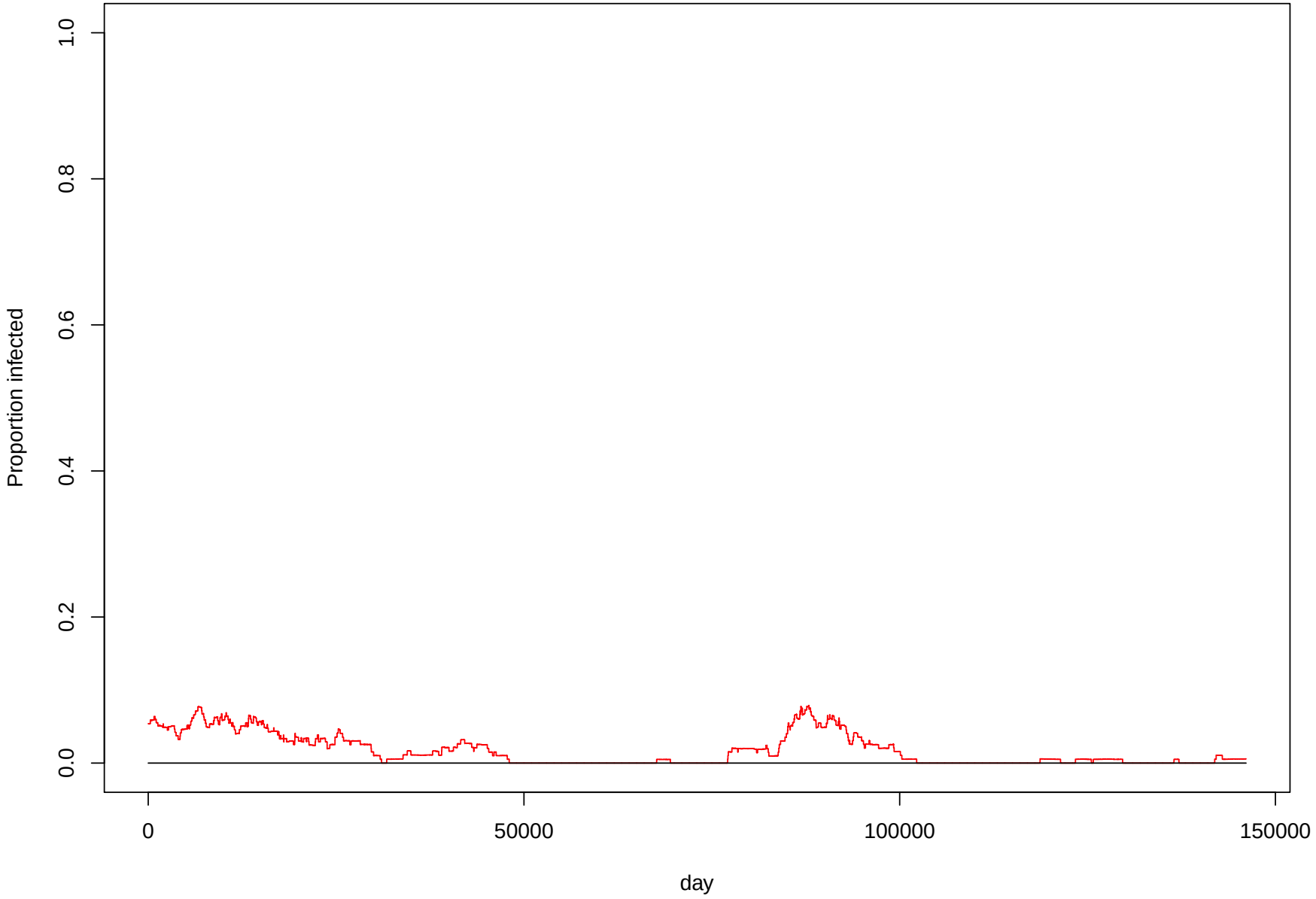
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