

Two Short Papers for the
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Borel's Paradox and Sensitivity Analysis

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Abstract

Borel's paradox has been identified as a concern in Bayesian synthesis analyses. Simple sensitivity analysis methods can be used to examine the degree to which the problem may occur in an application.

One criticism of the Bayesian synthesis method (Givens, 1993; Givens *et al.*, 1995a; Raftery *et al.*, 1995) is that inference from it may be subject to Borel's paradox, in the sense that the post-model distributions are not unique and depend on the particular functional form of parameters used in the model (Wolpert, 1995; Schweder and Hjort, 1995). It has been argued that the potential differences for any real application of the method should be investigated in detail (Bravington, 1995; Wolpert, 1995).

In this paper, we show how to investigate the potential effect of Borel's paradox using the reweighting methodology for sensitivity analysis introduced by Givens *et al.* (1994). Of course, the appropriate scope of sensitivity analysis may differ from one application to the next. For generic applications, Bravington (1995) gives a theoretical basis for determining the scope of sensitivity analysis, by considering "prior coherization". For the bowhead assessment in particular (Givens *et al.*, 1995a; IWC, 1995), the pre-model distribution of inputs is equivalent to a prior distribution, and the pre-model distribution of outputs is nearly a likelihood, since all of its elements except one are directly data-based, though not all are expressed in likelihood form in the Scientific Committee consensus analysis (IWC, 1995). Givens *et al.* (1995a,b) and Table 7 of IWC (1995) summarize the direct and indirect sources of evidence used for this assessment. Bowhead assessment is therefore quite close to a standard Bayesian analysis, for which Borel's paradox is not an issue. This also suggests a limited scope for sensitivity analysis.

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To see how sensitivity analysis can be useful, consider the simple population model $P_t = P_0 a^t$, where a is known and t is fixed, so that the model input is $\theta = P_0$ and the output is $\phi = P_t$. Suppose priors $p(\theta)$ and $p(\phi)$ exist for θ and ϕ , and a likelihood also exists for ϕ , based on data D . If the joint pre-model distribution is $p^{[\theta, \phi]}(\theta, \phi) = p(\theta)p(\phi | D) \propto p(\theta)p(\phi)p(D|\phi)$, then the post-model distribution is proportional to $p^{[\theta, \phi]}(\theta, a^t\theta)$ on the line where $\phi = \theta a^t$. If, instead, the model had been expressed in logarithmic terms as $\log P_t = \log P_0 + t \log a$, then the post-model distribution would have been proportional to $\theta p^{[\theta, \phi]}(\theta, a^t\theta)$ on the same line. This is the same post-model distribution we would obtain if the original pre-model distribution of θ was changed from $p(\theta)$ to $\theta p(\theta)$. In other words, changing the parameterization is operationally equivalent to changing the prior, so we can represent Borel's paradox as different priors with the same model parameterization, rather than the 'same' (i.e. equivalent but transformed) priors with different model parameterizations. Therefore, we can investigate the potential paradox by a standard sensitivity analysis. This holds for any set of transformations of (θ, ϕ) which is invertible. In the example considered above, we can use the reweighting method to investigate the extent to which Borel's paradox affects the results from the log model—precisely what was done by Raftery *et al.* (1995) in response to Wolpert's (1995) example.

The 'paradox' is that alternative model parameterizations can result in different post-model distributions. The question arises: how different can the answers be? Schweder and Hjort (1995) have demonstrated that the answers can be arbitrarily different, if one chooses to arbitrarily reparameterize the problem to such an end. However, it is worthwhile to also consider, for a given application, how much difference might occur in practice. This question can be answered through sensitivity analysis.

It is also worthwhile to consider how much reparameterization is reasonable in any given application. While any arbitrary reparameterization may be equally 'permissible' in a mathematical sense, not all reparameterizations are equally wise or reasonable for a given application; this is true not just for Bayesian synthesis, but also for many standard statistical procedures such as normal-based confidence intervals. For Bayesian synthesis, one way of evaluating the reasonableness of a reparameterization is to consider what it implies for pre-model distributions in the original parameterization. In the above example, the log reparameterization implied that the new pre-model distribution of θ was proportional to $\theta p(\theta)$. This modified pre-model distribution might also be a tolerable representation of initial uncertainty about θ , at least in the range supported by the other data, D , but for more extreme transformations, the new pre-model distribution would be very different from the original $p(\theta)$. For

example, if all variables were raised to the tenth power, the new pre-model probability of θ being above its original pre-model median would generally be very different from 0.5 after transforming. Box and Tiao (1973) discuss this issue in the context of setting locally uniform priors to represent minimal prior information: “for an extreme transformation such as θ^{10} ... a rational experimenter would not agree to employ a uniform prior. ... Once he understood the implication of taking a constant prior distribution for this extreme transformation, he would be unwilling to accept it.” Bravington (1995) also argues some transformations are too extreme to be reasonable. Thus, although we support the use of sensitivity analysis to investigate Borel’s paradox, we also argue that the scope of such investigations should be limited.

Bravington (1995) suggests a related use of sensitivity analysis, to investigate the effect of labeling different sets of parameters as ‘inputs’ and ‘outputs’. In addition to investigating Borel’s paradox, there may of course be other sensitivity analyses to consider, for example if the interpretation of some of the data was in question.

Together, these sensitivity analyses should shed considerable light on whether Borel’s paradox is important for any particular application. If the analyses show only modest sensitivity, then there is little reason for concern. The reweighting tools needed to conduct these sensitivity analyses (Givens *et al.*, 1994) are readily available and easily implemented. The results, being alternative versions of post-model distributions, are straightforward to interpret.

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Assessment of the Bering-Chukchi-Beaufort Seas Stock of Bowhead Whales with Updated Abundance and Trend Data

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Abstract

In its 1994 assessment of the Bering-Chukchi-Beaufort Seas stock of bowhead whales, the Scientific Committee agreed to use a stock abundance estimate based on data from 1988 because the estimate from the more recent 1993 data was preliminary at that time. Analyses of these recent data have now been revised to produce updated abundance and trend estimates. We present here the results of an assessment like that completed by the SC in 1994, but using these revised abundance and trend estimates.

1 INTRODUCTION

The Scientific Committee (SC) of the International Whaling Commission (IWC) used the Bayesian synthesis assessment method (Givens, 1993; Givens *et al.*, 1995a; Raftery *et al.*, 1995a) in its 1994 assessment of the Bering-Chukchi-Beaufort Seas stock of bowhead whales (IWC, 1995). Although a preliminary analysis of data on 1993 stock abundance was available, the SC chose not to include this analysis in their assessment because not all relevant data had yet been included in the analysis. Instead, the SC chose to rely on the most recent complete analysis, which was based on data from 1988. This decision was based on SC consideration of both the preliminary analysis for 1993 (Raftery and Zeh, 1994) and the previous work for 1988 (Raftery and Zeh, 1991; Raftery and Zeh, 1993).

Analysis of the 1993 data has been substantially advanced during the past year (Zeh *et al.*, 1995). In this paper, we present a bowhead assessment which uses the 1993 analysis in place of the 1988 analysis.

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2 METHOD

The 1994 SC bowhead assessment was based on SC consensus summaries of the data and evidence about a large number of biological and population parameters including survival rates, age of sexual maturity, fertility rates, and population abundance, trend, and age structure. Evidence about every parameter used in the assessment was summarized by a probability distribution. These consensus SC distributions are shown in Table 7 of the 1994 SC report (IWC, 1995).

The revised analysis of the 1993 abundance data affects the distributions for two variables: current population size and recent rate of population increase. In 1994, the SC chose to consider 1988 as the 'current' population year and used the abundance estimate for that year. Here, we consider 1993 as the 'current' population year and use the revised abundance estimate from Zeh *et al.* (1995), which provides a posterior mode of 8,200 whales, with 95% posterior probability interval of (7,300, 9,200), compared to the 1988 values of 7,500 and (6,400, 9,200). The revision of this abundance estimate also affects the estimated average annual rate of population increase from 1978–1993. We replace the distribution used by the SC (Raftery *et al.*, 1995b) with the results of an identical analysis (Zeh *et al.*, 1995) using the updated data, which provides an estimate of 3.2% and a 95% interval of (1.4%, 5.1%), compared to the SC values of 3.1% and (1.4%, 4.7%). All other elements of the 1994 SC assessment are left unchanged.

3 RESULTS

Using the evidence for population abundance and trend from the revised analysis of 1993 data along with all other evidence as agreed by the 1994 SC, we obtain post-model medians and 95% probability intervals shown in Table 1 for key quantities of interest. The results of the 1994 SC analysis are also shown in this table. Also included in Table 1 are results for a statistic Q , which Givens *et al.* (1995b) introduce as a catch limit which implements the existing Aboriginal Management Scheme for subsistence whaling for bowheads.

These results show that the revised assessment with updated abundance and trend data is very similar to the 1994 SC assessment. These results show no apparent reason for the SC to reconsider its 1994 bowhead subsistence whaling advice, although the SC may wish to update its estimates of quantities related to bowhead stock assessment.

Table 1: Estimates of key management quantities from the 1994 SC assessment and the revised analysis using updated results. Shown are post-model medians and 95% probability intervals. The 5th percentiles of replacement yield and Q are also given. The SC results for total depletion, MSY, and Q are obtained from a replication of the 1994 SC analysis (Givens, 1995), rather than the SC report.

Quantity	Revised	1994 SC
Initial stock size	12,487 (10,675, 18,166)	12,599 (10,945, 17,431)
$MSY R_{(mat)}$	4.46% (1.71%, 6.33%)	4.44% (1.93%, 6.65%)
1978-93 Average annual rate of increase	2.34% (0.78%, 3.21%)	2.31% (0.92%, 3.44%)
1993 Mature depletion	0.35 (0.29, 0.42)	0.36 (0.27, 0.44)
1993 Total depletion	0.66 (0.45, 0.80)	0.60 (0.41, 0.76)
1993 Replacement yield	213 (5%: 108) (90, 295)	199 (5%: 104) (97, 300)
Q	212 (5%: 107) (86, 294)	203 (5%: 102) (89, 308)

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