WinBUGS Example 1: Lip cancer

Consider the areal data disease mapping model:

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- The \( x_i \) are explanatory spatial covariates; typically \( \beta \) has a flat prior.
- The \( \theta_i \) capture heterogeneity among the regions via

\[ \theta_i \overset{iid}{\sim} N(0, 1/\tau_h), \]
and the $\phi_i$ capture regional clustering via a conditionally autoregressive (CAR) prior,

$$
\phi_i \mid \phi_{j \neq i} \sim N(\bar{\phi}_i, \frac{1}{\tau c m_i})
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where $\bar{\phi}_i = m_i^{-1} \sum_{j \in \partial_i} \phi_j$, $\partial_i$ is the set of “neighbors” of region $i$, and $m_i$ is the number of these neighbors.
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- Making the reparametrization from $(\theta, \phi)$ to $(\theta, \xi)$, we have the joint posterior

$$p(\theta, \xi \mid y) \propto L(\xi; y)p(\theta)p(\xi - \theta).$$
This means that

\[ p(\theta_i \mid \theta_j \neq i, \xi, y) \propto p(\theta_i) p(\xi_i - \theta_i \mid \{\xi_j - \theta_j\}_{j \neq i}) . \]

Since this distribution is free of the data \( y \), the \( \theta_i \) are Bayesianly unidentified (and so are the \( \phi_i \)).
WinBUGS Example 1: Lip cancer

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\[ \text{BUT: this does not preclude Bayesian learning about } \theta_i; \text{ this would instead require} \]

\[ p(\theta_i \mid y) = p(\theta_i) . \]

[Stronger condition: data have no impact on the marginal (not conditional) posterior.]
Dilemma: Though unidentified, the $\theta_i$ and $\phi_i$ are interesting in their own right, as is

$$\alpha = \frac{sd(\phi)}{sd(\theta) + sd(\phi)},$$

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Tricky to specify a “fair” prior balance between heterogeneity and clustering (e.g., one for which $\alpha \approx 1/2$) since $\theta_i$ prior is specified marginally while the $\phi_i$ prior is specified conditionally!
WinBUGS Example 1: Lip cancer

⋆ left panel: \( \frac{100Y_i}{E_i} \) (SMR), where \( Y_i \) = observed and \( E_i \) = expected cases for \( I = 56 \) districts, 1975–1980
WinBUGS Example 1: Lip cancer

⋆ left panel: $100\frac{Y_i}{E_i}$ (SMR), where $Y_i =$ observed and $E_i =$ expected cases for $I = 56$ districts, 1975–1980

⋆ right panel: $x_i$, % of the population engaged in agriculture, fishing or forestry (AFF covariate)
WinBUGS Example 1: Lip cancer

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☆ we also have: a variety of vague, proper, and arguably “fair” priors for $\tau_c$ and $\tau_h$
WinBUGS Example 1: Lip cancer

For actual WinBUGS code, see:
http://www.biostat.umn.edu/~brad/data/Lipsbrad.odc

Results:

AFF covariate appears significantly different from 0 under all 3 priors, although convergence is very slow
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- Excess variability in the data is mostly due to clustering \( E(\alpha|y) > .50 \), but the posterior distribution for \( \alpha \) does not seem robust to changes in the prior.
- Convergence for the \( \xi_i \) (reasonably well-identified) is rapid; convergence for the \( \mu_i \) (not shown) is virtually immediate.
WinBUGS Example 1: Lip cancer

Posterior and MCMC convergence summaries:

<table>
<thead>
<tr>
<th>Priors for $\tau_c$, $\tau_h$</th>
<th>Posterior for $\alpha$</th>
<th>Posterior for $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>sd</td>
</tr>
<tr>
<td>G(1.0, 1.0), G(3.2761, 1.81)</td>
<td>.57</td>
<td>.058</td>
</tr>
<tr>
<td>G(.1, .1), G(.32761, .181)</td>
<td>.65</td>
<td>.073</td>
</tr>
<tr>
<td>G(.1, .1), G(.001, .001)</td>
<td>.82</td>
<td>.10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Priors for $\tau_c$, $\tau_h$</th>
<th>Posterior for $\xi_1$</th>
<th>Posterior for $\xi_{56}$</th>
</tr>
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<td>.40</td>
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<tr>
<td>G(.1, .1), G(.32761, .181)</td>
<td>.89</td>
<td>.36</td>
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WinBUGS Example 2: Home prices

Here we illustrate a non-Gaussian model for point-referenced spatial data:

Data: Observations are home values (based on recent real estate sales) at 50 locations in Baton Rouge, Louisiana, USA.
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Data: Observations are home values (based on recent real estate sales) at 50 locations in Baton Rouge, Louisiana, USA.

The response $Y(s)$ is a binary variable, with

$$Y(s) = \begin{cases} 
1 & \text{if price is “high” (above the median)} \\
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WinBUGS Example 2: Home prices

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\]

- Observed covariates include the house’s **age** and total living area
WinBUGS Example 2: Home prices

We fit a generalized linear model where
\[ Y(s_i) \sim Bernoulli(p(s_i)), \quad \text{logit}(p(s_i)) = x^T(s_i)\beta + w(s_i) \]
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Assume vague priors for \( \beta \), a Uniform\((0, 10)\) prior for \( \phi \), and an Inverse Gamma\((0.1, 0.1)\) prior for \( \sigma^2 \).
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- The WinBUGS code and data for this example are at www.biostat.umn.edu/~brad/data/BatonRougebinary.bug:

```
for (i in 1:N) {
    Y[i] ~ dbern(p[i])
    logit(p[i]) <- w[i]
for (i in 1:3) beta[i] ~ dnorm(0.0,0.001)
w[1:N] ~ spatial.exp(mu[], x[], y[], spat.prec, phi, 1)
phi ~ dunif(0.1,10)
spat.prec ~ dgamma(0.1, 0.1)
sigmasq <- 1/spat.prec
```
WinBUGS Example 2: Home prices

Use `image` and `contour` on $w_i$ posterior medians in R
WinBUGS Example 2: Home prices

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- negative residuals (i.e., lower prices) in the north;
  positive residuals (i.e., higher prices) in the south
WinBUGS Example 2: Home prices

- Use `image` and `contour` on $w_i$ posterior medians in R
- **negative** residuals (i.e., **lower** prices) in the north;
  **positive** residuals (i.e., **higher** prices) in the south
- smooth flat stretches across the central parts;
  downward slopes toward the north and southeast.
## WinBUGS Example 2: Home prices

Parameter estimates (posterior medians and upper and lower .025 points):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>50%</th>
<th>(2.5%, 97.5%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$ (intercept)</td>
<td>$-1.096$</td>
<td>($-4.198, 0.4305$)</td>
</tr>
<tr>
<td>$\beta_2$ (living area)</td>
<td>$0.659$</td>
<td>($-0.091, 2.254$)</td>
</tr>
<tr>
<td>$\beta_3$ (age)</td>
<td>$0.009615$</td>
<td>($-0.8653, 0.7235$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$5.79$</td>
<td>($1.236, 9.765$)</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>$1.38$</td>
<td>($0.1821, 6.889$)</td>
</tr>
</tbody>
</table>

The covariate effects are generally uninteresting, though living area seems to have a marginally significant effect on price class.