A Semiparametric Stochastic Mixed Model for Increment-Averaged Data with Application to Carbon Sequestration in Agricultural Soils

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Space-**T**ime **A**quatic **R**esources **M**odeling & **A**nalysis **P**rogram

- **STARMAP:** EPA-funded STAR grant
  - study probability surveys of aquatic resources
  - CSU focuses on modeling side
  - partners with sister program at OSU (design side)
- Um, agricultural soils $\neq$ aquatic resources?
  - spin-off from other work on nonparametric survey regression estimation

Greenhouse Effect

- Solar energy transmitted to earth as visible and ultraviolet radiation

	Atmosphere	Surface
Reflected	25%	5%
Absorbed	25%	45%

- Radiation absorbed by surface gets re-radiated as infrared
- **Greenhouse Gases (GHGs)**
  - pass visible and UV, but trap infrared
  - include water vapor, CO_2, methane, nitrous oxide, others

Kyoto Protocol

- December 1997 meeting in Kyoto, Japan
- Resulted in Kyoto Protocol
  - signed by over 170 nations, including US
- Binding commitment for US to reduce emissions of six GHGs
  - reduce 7% from 1990 levels during 2008–2012
- US never ratified Kyoto
  - no commitment by developing nations
  - potentially large economic impact

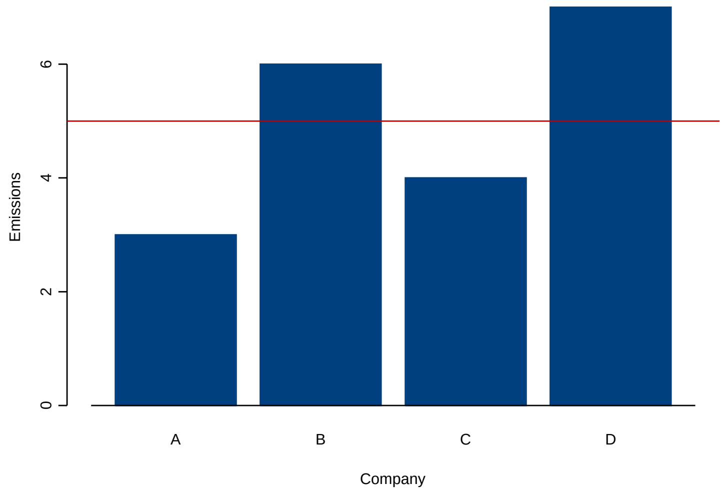
Chicago Climate Exchange

- www.chicagoclimatex.com
- Participants set voluntary limits on GHG emissions
  - legally binding commitments for reductions

	Commitments			
Baseline	2003	2004	2005	2006
avg. 1998–2001	-1%	-2%	-3%	-4%

Why Submit to Binding Commitments?

- Creates a "cap-and-trade" market
  - can make own reductions
  - can buy credits from others that have extra reductions to sell



Why Participate?

- Participants include Ford, Motorola, DuPont, City of Chicago, Waste Management, . . .
- Reduce long-term costs of carbon reduction
- Get financial benefits (e.g., reduced energy costs)
- Enhance environmental leadership reputation
- Build trading skills / help create market rules

Examples of GHG Mitigation and Offset Projects

- Renewal energy systems
  - wind power, solar power
- Energy efficient process innovations
- Recovery / use of landfill and agricultural methane
- Carbon sequestration
  - tree biomass
  - agricultural soils: **no-till agriculture**

Agriculture 001: Tillage

- Traditional Tillage:
  - after harvest, field contains crop residues
  - tillage turns over the soil to bury residues
  - often repeated several times prior to planting
- Conservation Tillage:
  - Reduced-Till: limited tillage; substantial crop residues on surface
  - No-Till (**NT**): doesn't use tillage; all crop residues left on surface
  - Economically feasible due to new technology

Advantages of No-Till Management

- NT results in lower production costs
  - fewer management steps
  - cheaper, lower horsepower tractors
- NT leaves crop residue on soil surface
  - reduces soil loss due to wind and water erosion
  - reduces flow of sediments, nutrients, and pesticides into surface waters
  - improves soil fertility; enhances soil organic matter
  - **sequesters carbon**

Carbon Sequestration via No-Till

- Original soil carbon content has been reduced 50% since invention of the steel plow
  - aeration due to tillage leads to faster decomposition
  - decomposition releases CO_2 to atmosphere
- No-till significantly slows decomposition
  - widespread adoption could restore original carbon within 40 years
  - could cut projected growth in US CO_2 emissions by 20%

Show Me the Money

- Carbon as a cash crop?
- Government could subsidize switch to no-till
- Corporations faced with caps on GHG emissions could buy **carbon credits**
  - some Canadian companies have already paid Iowa farmers for carbon credits
- Estimates of value range from \$4–\$40 per acre

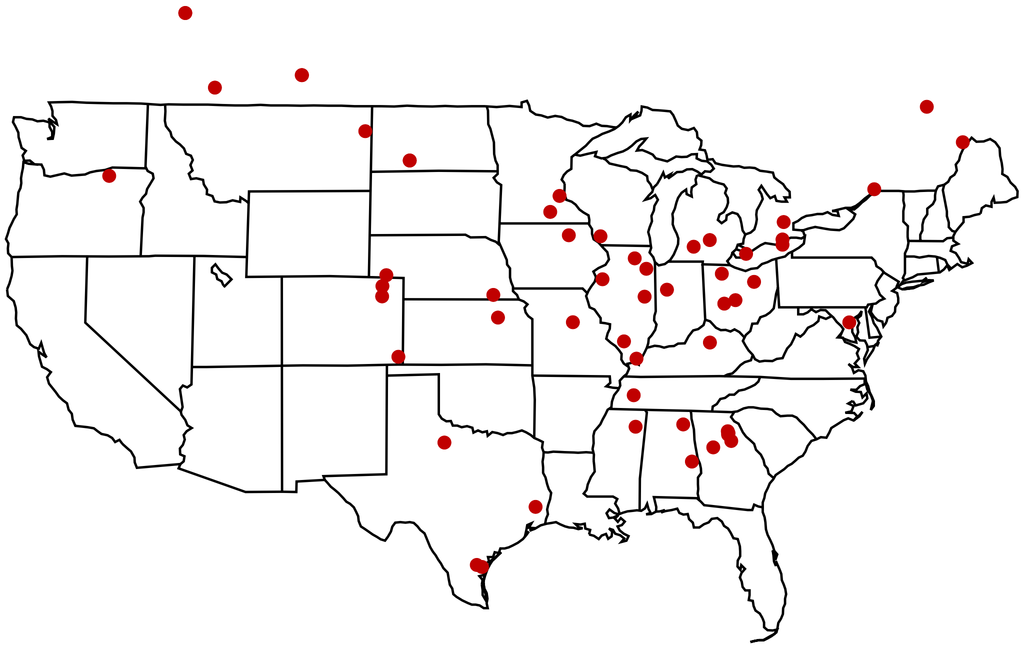
Key Question for Carbon Credits

- How much carbon is sequestered in switching to no-till?
- Several studies have compared no-till with traditional tillage on paired fields
- Measure carbon difference after one or more years since management change:

$$Y = (\text{no-till carbon}) - (\text{traditional tillage carbon})$$

Available North American Studies

- 63 studies compare tillage types on paired fields



Soil Core Samples

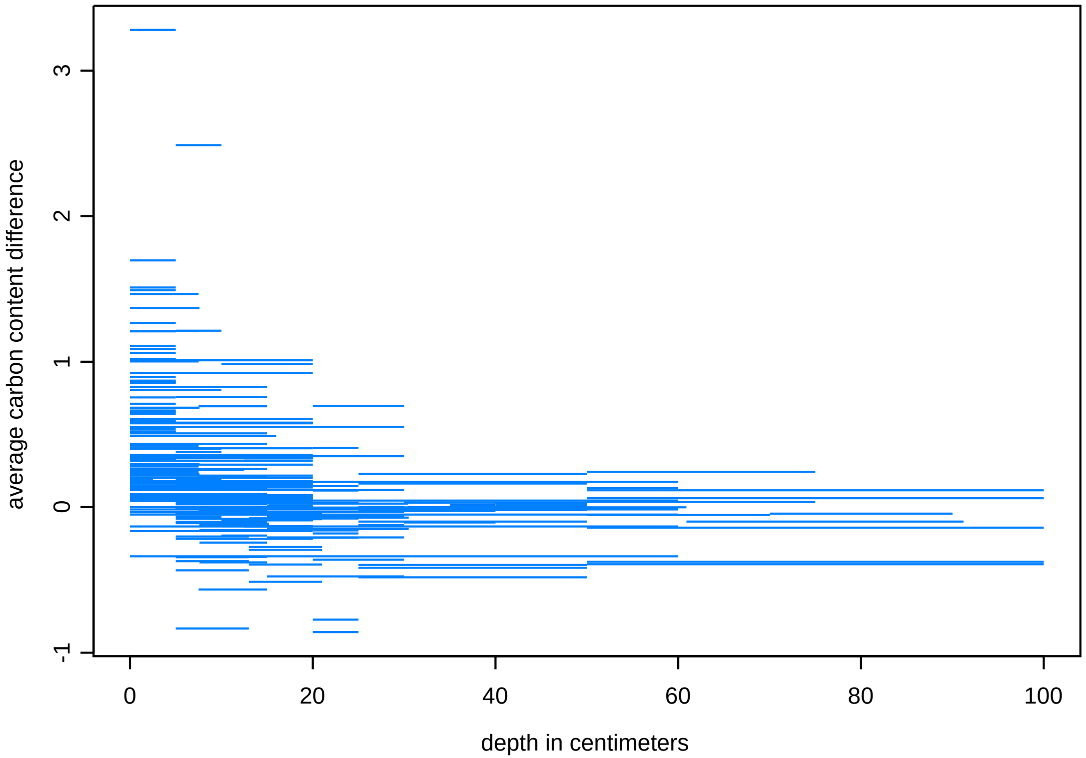
- Use probe to select one or more cores
- Separate cores into increments
  - may be fixed increments (e.g., 0–15 cm, 15–30 cm)
  - may be determined by soil profile (e.g., plow layer, A horizon, B horizon, C horizon)
- Mix matching increments in a bucket
- Bag a subsample and send to the lab

Aside on Core Data

- Like a time series, with depth replacing time
  - specifically, a **flow**: defined on interval (income, expenditure, precipitation)
  - not a **stock**: instantaneous (interest rate, temp)
- Other "cores":
  - ice cores: increment represents one year of snowfall
  - vertical ozone profiles
  - like time series, tend to be many, regularly spaced, relatively narrow increments
- Not true for soil cores!

Increment-Averaged Data

- Difference in metric tons C ha^{-1} vs. depth



Ad Hoc Methods: Midpoint Assignment

- True model is

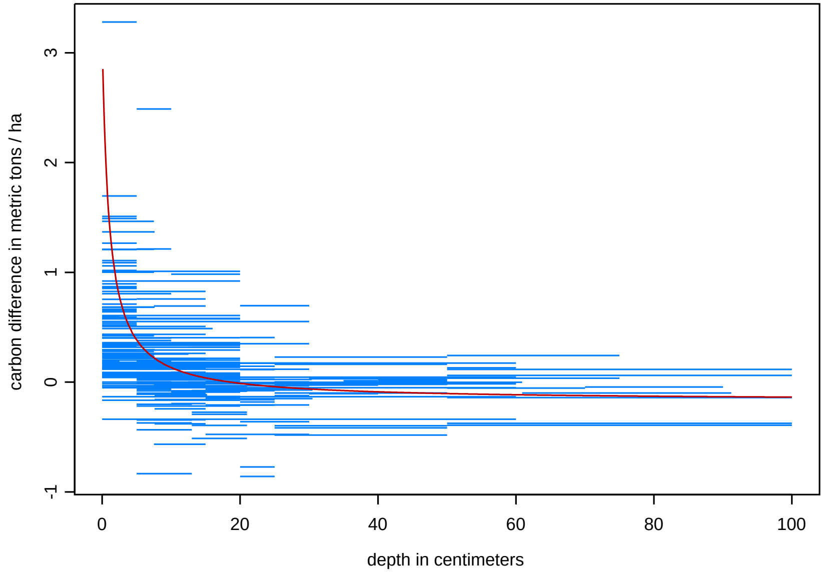
$$\begin{aligned} Y_{ij} &= \frac{1}{d_{ij} - d_{i,j-1}} \int_{d_{i,j-1}}^{d_{ij}} \sum_k \alpha_k g_k(t)\, dt + \epsilon_{ij} \\ &= \sum_k \alpha_k \frac{1}{d_{ij} - d_{i,j-1}} \int_{d_{i,j-1}}^{d_{ij}} g_k(t)\, dt + \epsilon_{ij} \\ &= \sum_k \alpha_k g_k(t^*_{ijk}) + \epsilon_{ij} \text{ where } t^*_{ijk} \in (d_{i,j-1}, d_{ij}) \end{aligned}$$

- But we regress on $g_k(m_{ij}) \neq g_k(t^*_{ijk})$: measurement error problem
- Least squares estimators are biased and inconsistent

Do We Need to Worry?

- Simulate from the fitted model

$$Y_{ij} = \frac{1}{d_{ij} - d_{i,j-1}} \int_{d_{i,j-1}}^{d_{ij}} (\alpha_0 + \alpha_1/(1+t))\ dt + \epsilon_{ij}$$



Simulation Results

- Simulate from true model, then estimate with midpoint assignment
  - use actual increments from data set
  - resample if necessary

sample size	**reps**	$\boldsymbol{\alpha_0 = -0.17}$	$\boldsymbol{\alpha_1 = 3.32}$
218	10,000	$\mathrm{E}[\hat{\alpha}_0] = -0.17$	$\mathrm{E}[\hat{\alpha}_1] = 4.21$
10,000	1	$\hat{\alpha}_0 = -0.17$	$\hat{\alpha}_1 = 4.18$

- Over 25% relative bias in slope estimate

Ad Hoc Methods: Adjustment of Increments

- Study 1 measures

$$Y_{11} = \text{carbon stock change over increment 0–15 cm}$$
$$Y_{12} = \text{carbon stock change over increment 15–30 cm}$$

- Study 2 measures

$$Y_{21} = \text{carbon stock change over increment 0–50 cm}$$

- "Adjustment" of Y-values

$$Y_1^* := Y_{11} + Y_{12} \text{ represents 0–30 cm}$$
$$Y_2^* := \frac{30}{50} Y_{21} \text{ represents 0–30 cm}$$

Ad Hoc Methods for Increments

- Midpoint assignment leads to bias, inconsistency
- "Adjustment" leads to loss of information, likely bias
- One final method: simply drop studies with non-matching increments
  - obvious loss of information
- Need to recognize the increment nature of the data

Key Data Features

- Increment averaging
  - irregular, wide
  - difficult to specify parametric model
- Within-core dependence
  - increments within same core may be correlated
- Other effects
  - time since change to no-till
  - climate regime
  - soil type

Semiparametric Stochastic Mixed Model: Longitudinal

- Zhang, Lin, Raz, Sowers (1998) *JASA*:

$$Y_{ij} = \boldsymbol{X}_{ij}^T \boldsymbol{\beta} + g(t_{ij}) + \boldsymbol{Z}_{ij}^T \boldsymbol{b}_i + U_i(t_{ij}) + \epsilon_{ij}$$

  - $\boldsymbol{\beta}$: p unknown regression coefficients
  - $\boldsymbol{X}_{ij}, \boldsymbol{Z}_{ij}$: known covariates
  - $g(t)$: twice-differentiable function of time
  - $\boldsymbol{b}_i$: independent q-vectors of random effects
  - $U_i(t)$: independent stochastic processes
  - ϵ_{ij}: independent errors
- Does not handle increment averages

Semiparametric Stochastic Mixed Model: Increments

- Increment average in ith core, jth increment

$$Y_{ij} = \boldsymbol{X}_{ij}^T \boldsymbol{\beta} + \frac{1}{d_{ij} - d_{i,j-1}} \int_{d_{i,j-1}}^{d_{ij}} g(t)\, dt + \boldsymbol{Z}_{ij}^T \boldsymbol{b}_i + U_{ij} + \epsilon_{ij}$$

  – $\boldsymbol{\beta}$: p unknown regression coefficients
  – $\boldsymbol{X}_{ij}, \boldsymbol{Z}_{ij}$: known covariates
  – $g(t)$: twice-differentiable function of depth
  – $\boldsymbol{b}_i$: independent q-vectors of random effects
  – U_{ij}: increment-averaged stochastic process
  – ϵ_{ij}: independent errors

Assumptions for Semiparametric Stochastic Mixed Model

- Define

$$\boldsymbol{U}_i = \left(U_{i1}, U_{i2}, \ldots, U_{in_i}\right)^T$$

- Assume
  - $\boldsymbol{U}_i$ are independent normal$(\mathbf{0}, \boldsymbol{\Gamma}_i)$ where $\boldsymbol{\Gamma}_i = \text{cov}\left(\boldsymbol{U}_i, \boldsymbol{U}_i; \boldsymbol{\xi}\right)$
  - $\boldsymbol{b}_i$ are independent normal$(\mathbf{0}, \boldsymbol{D}(\boldsymbol{\phi}))$ where $\boldsymbol{D}$ is a positive definite matrix
  - ϵ_{ij} are independent normal$(0, \sigma^2)$
  - $\boldsymbol{b}_i$, $\boldsymbol{U}_i$, and ϵ_{ij} are mutually independent

Integrated Stochastic Process Specification

- Increment-averaged form:

$$U_{ij} = \frac{1}{d_{ij} - d_{i,j-1}} \int_{d_{i,j-1}}^{d_{ij}} U_i(t)\, dt,$$

  where $U_i(t)$ are independent, mean zero, Gaussian stochastic processes

- Choices for instantaneous process:
  - Wiener process
  - integrated Wiener process
  - non-homogeneous Ornstein-Uhlenbeck process

Nonhomogeneous Ornstein-Uhlenbeck Process

- Covariance structure for instantaneous process:

$$\begin{aligned} \operatorname{var}(U_i(t)) &= \xi(t) = \exp\{\xi_0 + \xi_1 t\} \\ \operatorname{corr}(U_i(s), U_i(t)) &= \exp\{-\alpha|s-t|\} \end{aligned}$$

- Covariances for increment-averaged process:

$$\begin{aligned} &\operatorname{cov}(U_{ij}, U_{ik}) \\ &\quad = \frac{e^{\xi_0}}{(d_{ij} - d_{i,j-1})(d_{ik} - d_{i,k-1})} \\ &\quad \times \frac{e^{d_{ij}(\frac{\xi_1}{2}+\alpha)} - e^{d_{i,j-1}(\frac{\xi_1}{2}+\alpha)}}{\frac{\xi_1}{2}+\alpha} \frac{e^{d_{ik}(\frac{\xi_1}{2}-\alpha)} - e^{d_{i,k-1}(\frac{\xi_1}{2}-\alpha)}}{\frac{\xi_1}{2}-\alpha} \end{aligned}$$

Integrated Nonparametric Function Specification

- "Parameter" $g(t)$ is infinite-dimensional
- Assume $g(t)$ is **natural cubic spline**
  - knots at distinct right-hand endpoints
  - cubic polynomial between knots
  - g, g', g'' continuous at knots (hence everywhere)
  - linear before first knot, after last knot

Splitting Up the Integral

- Distinct right-hand endpoints $= \boldsymbol{t} = (t_1, t_2, \ldots, t_r)^T$
- Increment total:

$$\begin{aligned}
&\int_{d_1}^{d_2} g(t)\, dt \\
&= \int_{t_k=d_1}^{t_{k+1}} g(t)\, dt + \int_{t_{k+1}}^{t_{k+2}} g(t)\, dt + \cdots + \int_{t_{k+p-1}}^{t_{k+p}=d_2} g(t)\, dt \\
&= (0, \ldots, 0, 1, \ldots, 1, 0, \ldots, 0)\boldsymbol{G}
\end{aligned}$$

for some non-negative integer k and some positive integer p, where

$$\boldsymbol{G} = \left(\int_{t_0}^{t_1} g(t)\, dt, \int_{t_1}^{t_2} g(t)\, dt, \ldots, \int_{t_{r-1}}^{t_r} g(t)\, dt \right)^T$$

Value-Second Derivative Representation

- Define $g_i = g(t_i)$ and $\gamma_i = g''(t_i)$ for $i = 1, 2, \ldots, r$ $(\gamma_1 = \gamma_r = 0)$
- For $h_i = t_{i+1} - t_i$, we have

$$G_1 = \int_0^{t_1} g(t)\,dt = \left(t_1 + \frac{t_1^2}{2h_1}\right) g_1 - \frac{t_1^2}{2h_1} g_2 + \frac{h_1}{6}\frac{t_1^2}{2}\gamma_2$$

and

$$G_{i+1} = \int_{t_i}^{t_{i+1}} g(t)\,dt = \frac{h_i}{2}(g_i + g_{i+1}) - \frac{h_i^3}{24}(\gamma_i + \gamma_{i+1})$$

for $i = 1, 2, \ldots, r-1$

Ah, Linear

- So $\boldsymbol{G}$ is a linear function of $\boldsymbol{g} = (g_1, \ldots, g_r)^T$ and $\boldsymbol{\gamma} = (\gamma_2, \ldots, \gamma_{r-1})^T$:

$$\boldsymbol{G} = [\boldsymbol{A}_1, \boldsymbol{A}_2] \begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{\gamma} \end{bmatrix} = [\boldsymbol{A}_1, \boldsymbol{A}_2] \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{R}^{-1}\boldsymbol{Q}^T \end{bmatrix} \boldsymbol{g},$$

  where $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, $\boldsymbol{R}$ and $\boldsymbol{Q}$ are known matrices (depending only on $\{h_i\}$)

- In fact, $\boldsymbol{G}$ is linear function of $\boldsymbol{g}$ alone

Finite-Dimensional Model

- Integrals of infinite-dimensional parameter now reduced to r unknown constants:

$$
\begin{aligned}
\boldsymbol{Y}_i &= \boldsymbol{X}_i\boldsymbol{\beta} + \left[\frac{\int_{d_{i,j-1}}^{d_{ij}} g(t)\,dt}{d_{ij} - d_{i,j-1}}\right]_{j=1}^{n_i} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{U}_i + \boldsymbol{\epsilon}_i \\
&= \boldsymbol{X}_i\boldsymbol{\beta} + \begin{bmatrix} \boldsymbol{c}_{i1}^T \\ \vdots \\ \boldsymbol{c}_{in_i}^T \end{bmatrix} \boldsymbol{G} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{U}_i + \boldsymbol{\epsilon}_i \\
&= \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{N}_i\boldsymbol{g} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{U}_i + \boldsymbol{\epsilon}_i
\end{aligned}
$$

Likelihood

- Log-likelihood given covariance parameters $\boldsymbol{\phi}$, $\boldsymbol{\xi}$, and σ^2

$$\ell(\boldsymbol{\beta}, \boldsymbol{g}; \boldsymbol{Y}) = -\frac{1}{2}\log|\boldsymbol{V}|$$
$$-\frac{1}{2}(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{N}\boldsymbol{g})^T\boldsymbol{V}^{-1}(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{N}\boldsymbol{g})$$

where

– $\boldsymbol{V} = \mathrm{diag}\{\boldsymbol{V}_1, \boldsymbol{V}_2, \ldots, \boldsymbol{V}_m\}$

– $\boldsymbol{V}_i = \boldsymbol{Z}_i\boldsymbol{D}(\boldsymbol{\phi})\boldsymbol{Z}_i^T + \boldsymbol{\Gamma}_i + \sigma^2\boldsymbol{I}$

Penalized Likelihood

- Log-likelihood given covariance parameters $\boldsymbol{\phi}$, $\boldsymbol{\xi}$, and σ^2

$$\ell(\boldsymbol{\beta}, \boldsymbol{g}; \boldsymbol{Y}) = -\frac{1}{2}\log|\boldsymbol{V}|$$
$$-\frac{1}{2}(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{N}\boldsymbol{g})^T \boldsymbol{V}^{-1}(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{N}\boldsymbol{g})$$

- Penalized likelihood:

$$\ell(\boldsymbol{\beta}, \boldsymbol{g}; \boldsymbol{Y}) - \frac{\lambda}{2}\int_0^{t_r} \left[g''(t)\right]^2 \, dt$$

where $\lambda > 0$ controls smoothness

– $\lambda \to 0$ implies interpolating

– $\lambda \to \infty$ implies global regression

Penalized Likelihood, Continued

- Penalized likelihood can be rewritten:

  $$\ell(\boldsymbol{\beta}, \boldsymbol{g}; \boldsymbol{Y}) - \frac{\lambda}{2} \int_0^{t_r} \left[g''(t) \right]^2 \, dt = \ell(\boldsymbol{\beta}, \boldsymbol{g}; \boldsymbol{Y}) - \frac{\lambda}{2} \boldsymbol{g}^T \boldsymbol{K} \boldsymbol{g},$$

  where $\boldsymbol{K} = \boldsymbol{Q} \boldsymbol{R}^{-1} \boldsymbol{Q}^T$ is known

- Now differentiate with respect to $\boldsymbol{\beta}$ and $\boldsymbol{g}$

Maximum Penalized Likelihood Estimators

- Assume λ as well as $\boldsymbol{\phi}$, $\boldsymbol{\xi}$, and σ^2 are given
- MPLEs solve the following linear equations:

$$\begin{bmatrix} \boldsymbol{X}^T\boldsymbol{V}^{-1}\boldsymbol{X} & \boldsymbol{X}^T\boldsymbol{V}^{-1}\boldsymbol{N} \\ \boldsymbol{N}^T\boldsymbol{V}^{-1}\boldsymbol{X} & \boldsymbol{N}^T\boldsymbol{V}^{-1}\boldsymbol{N} + \lambda\boldsymbol{K} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta} \\ \boldsymbol{g} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}^T\boldsymbol{V}^{-1}\boldsymbol{Y} \\ \boldsymbol{N}^T\boldsymbol{V}^{-1}\boldsymbol{Y} \end{bmatrix}$$

- Unique solution?

Uniqueness of MPLEs

- By Theorem 4.1 of Green and Silverman (1994), unique solution if $[\boldsymbol{X}, \boldsymbol{N}\mathbf{1}, \boldsymbol{N}\boldsymbol{t}]$ has full column rank
- **Result:** If $\theta_0 \mathbf{1} + \theta_1 \boldsymbol{m} \notin \mathrm{Col}(\boldsymbol{X})$, where $\boldsymbol{m}$ is vector of increment midpoints, then

$$\begin{bmatrix} \boldsymbol{X}^T \boldsymbol{V}^{-1} \boldsymbol{X} & \boldsymbol{X}^T \boldsymbol{V}^{-1} \boldsymbol{N} \\ \boldsymbol{N}^T \boldsymbol{V}^{-1} \boldsymbol{X} & \boldsymbol{N}^T \boldsymbol{V}^{-1} \boldsymbol{N} + \lambda \boldsymbol{K} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta} \\ \boldsymbol{g} \end{bmatrix} = \begin{bmatrix} \boldsymbol{X}^T \boldsymbol{V}^{-1} \boldsymbol{Y} \\ \boldsymbol{N}^T \boldsymbol{V}^{-1} \boldsymbol{Y} \end{bmatrix}$$

  has a unique solution $(\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{g}}) = \mathrm{MPLE}$

Yet Another Representation: Linear Mixed Model

- From Green (1987) *ISR*, it turns out that

$$\boldsymbol{g} = (\mathbf{1}, \boldsymbol{t})\boldsymbol{\delta}_{2\times 1} + \boldsymbol{B}\boldsymbol{a}_{(r-2)\times 1},$$

  where $\boldsymbol{B}$ is a known matrix derived from $\boldsymbol{K}$

- Linear mixed model representation:

$$\begin{aligned}\boldsymbol{Y} &= [\boldsymbol{X}, \boldsymbol{N}\boldsymbol{T}]\begin{bmatrix}\boldsymbol{\beta} \\ \boldsymbol{\delta}\end{bmatrix} + \boldsymbol{N}\boldsymbol{B}\boldsymbol{a} + \boldsymbol{Z}\boldsymbol{b} + \boldsymbol{U} + \boldsymbol{\epsilon} \\ &= \boldsymbol{X}_*\boldsymbol{\beta}_* + \boldsymbol{N}\boldsymbol{B}\boldsymbol{a} + \boldsymbol{Z}\boldsymbol{b} + \boldsymbol{U} + \boldsymbol{\epsilon},\end{aligned}$$

  where $\boldsymbol{a}$ is normal$(\mathbf{0}, \tau\boldsymbol{I})$ and $\tau = 1/\lambda$

  – Tricky: *not* the data-generating mechanism

- "BLUE" of $\boldsymbol{\beta}$ and "BLUP" of $\boldsymbol{g}$ from mixed model = MPLEs

Estimation of Covariance and Smoothing Parameters

- Use REML = Restricted Maximum Likelihood
  - standard method for linear mixed models
- Define $\boldsymbol{V}_* = \tau(\boldsymbol{N}\boldsymbol{B})(\boldsymbol{N}\boldsymbol{B})^T + \boldsymbol{V}$
- REML criterion for $(\boldsymbol{\phi}^T, \boldsymbol{\xi}^T, \sigma^2, \tau)$ is

$$\begin{aligned}
&\ell_R((\boldsymbol{\phi}^T, \boldsymbol{\xi}^T, \sigma^2, \tau); \boldsymbol{Y}) \\
&\quad = -\frac{1}{2}\log|\boldsymbol{V}_*| - \frac{1}{2}\log|\boldsymbol{X}_*^T\boldsymbol{V}_*^{-1}\boldsymbol{X}_*| \\
&\qquad -\frac{1}{2}(\boldsymbol{Y} - \boldsymbol{X}_*\hat{\boldsymbol{\beta}}_*)^T\boldsymbol{V}_*^{-1}(\boldsymbol{Y} - \boldsymbol{X}_*\hat{\boldsymbol{\beta}}_*)
\end{aligned}$$

- Given REML estimates, MPLEs are immediate

Back to the Carbon Sequestration Data

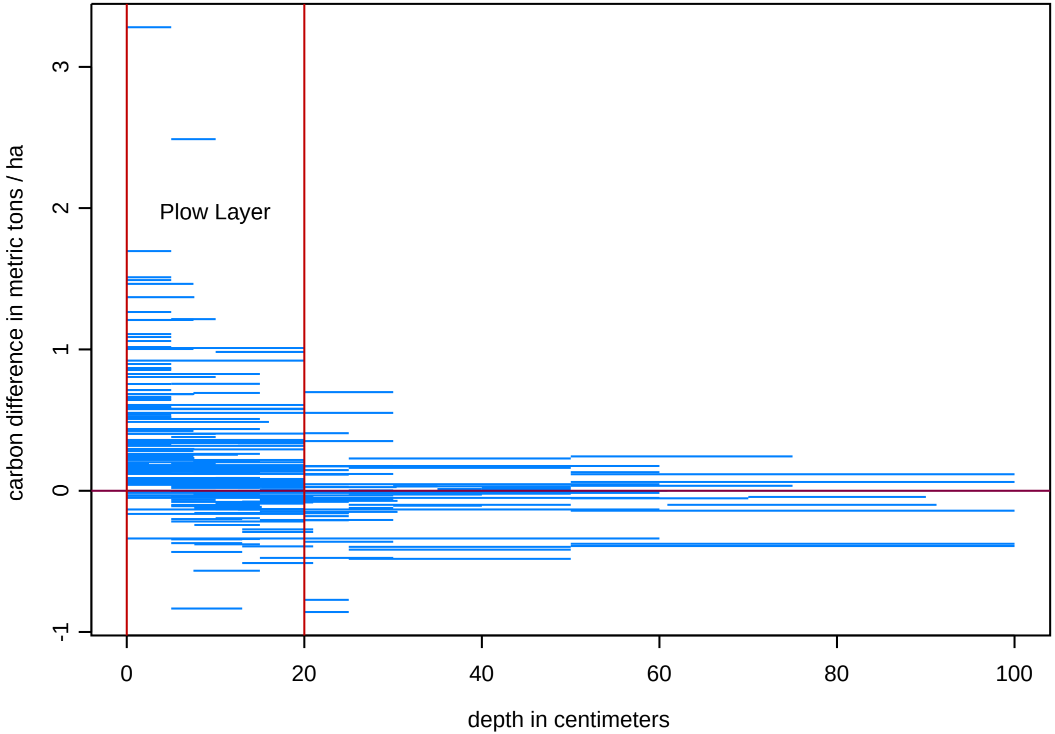
- Now have all the tools needed for inference in the carbon data set
- **Goals:**
  - identify important fixed effects
  - identify important sources of variation and correlation
  - estimate depth function
  - estimate expected carbon sequestered due to no-till:

$$\mathrm{IPCC} = \int_0^{30} \left(\boldsymbol{X}_{00}^T \boldsymbol{\beta} + g(t) \right) dt,$$

  where $\boldsymbol{X}_{00}$ = covariates at 20 years after management change

The Plow Layer

- Top 15–20 cm of agricultural soil



Model Specification

- Fixed effects:
  - wet/dry climate
  - aquic/non-aquic soils
  - years since management change
  - parametric or nonparametric year*depth interactions
- Random effect: soil core
- Stochastic process: non-homogeneous OU

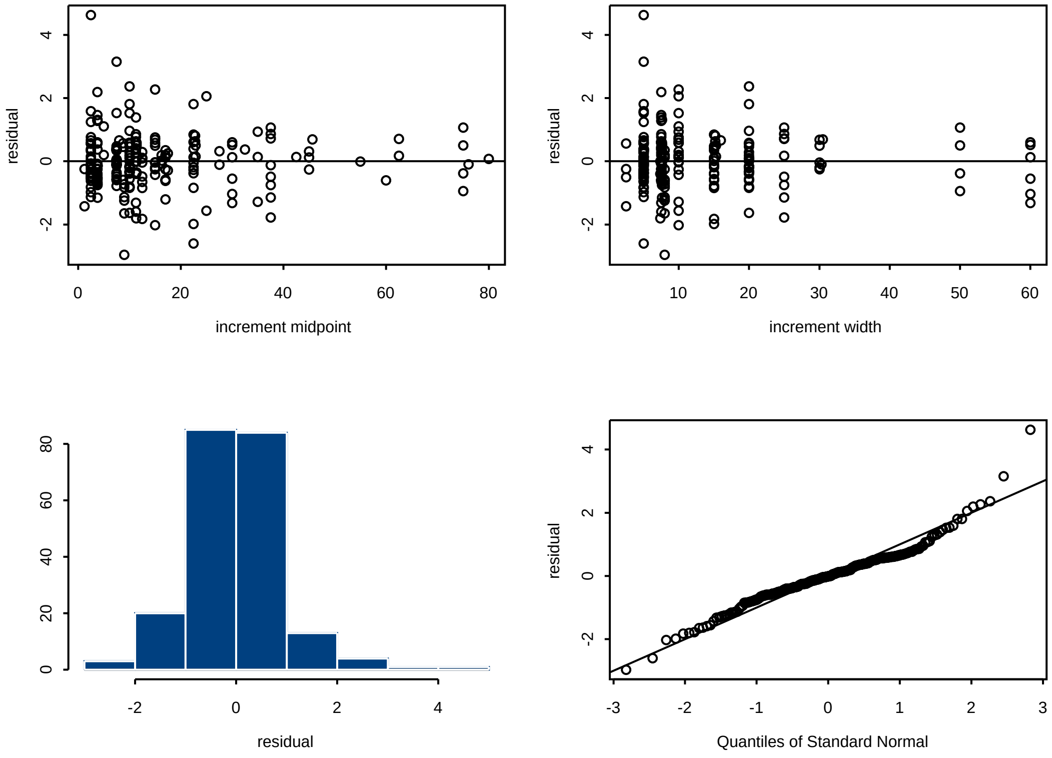
Some Results

- Estimation results from REML and MPLE:

	Model 1		Model 2	
Parameter	Estimate	Standard Error	Estimate	Standard Error
aquic	-0.1383	0.0587	-0.1371	0.0592
wet	0.1725	0.0593	0.1720	0.0596
years	0.0157	0.0047	0.0031	0.0032
year*depth	-0.0003	0.0001		
ϕ	0.0078	0.0077	0.0094	0.0093
σ^2	0.0073	0.0091	0.0064	0.0102
ξ_0	-0.8260	0.2368	-0.7911	0.2463
ξ_1	-0.0601	0.0205	-0.0643	0.0219
α	0.2607	0.0447	0.2693	0.0490
$\tau = \lambda^{-1}$	0.0326	0.0046	0.0189	0.0026
$\tau_x = \lambda_x^{-1}$			0.00005	0.000008

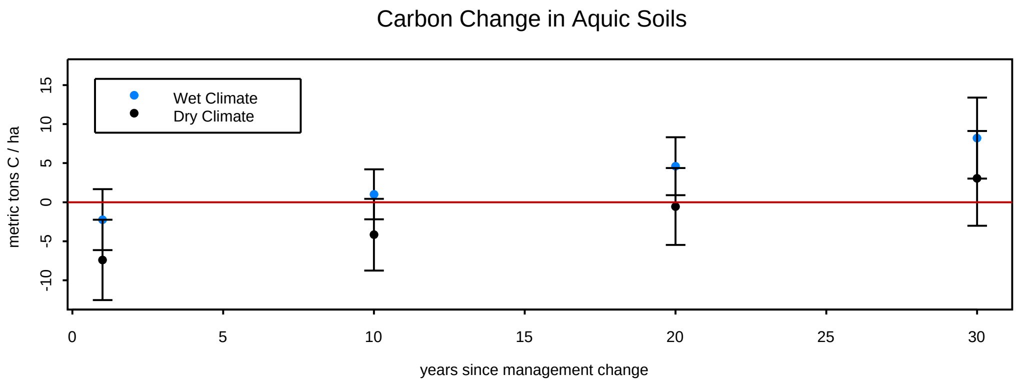
Residual Diagnostics

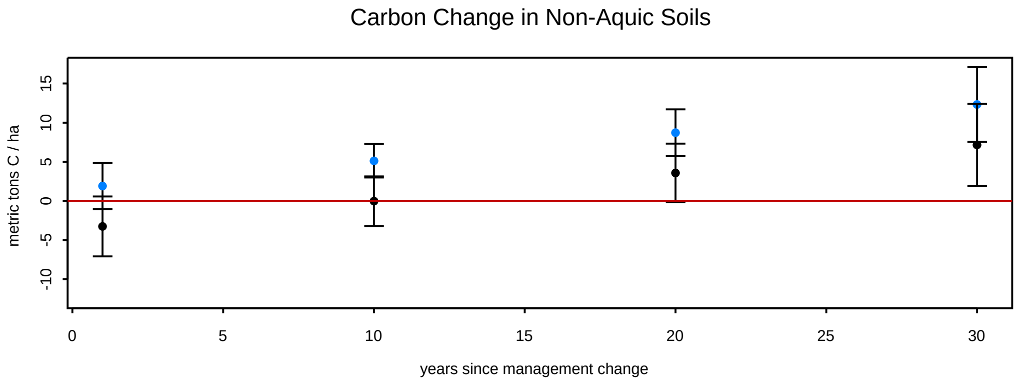
- Accounting for OU dependence structure



Intergovernmental Panel on Climate Change Integrals

- Estimated IPCC integrals





Summary and Future Research

- Increment averaging cannot be ignored in soil core data
- Semiparametric stochastic mixed model
  - flexibly models increment averages
  - handles fixed and random effects
  - fits in standard linear mixed model framework
- Further work:
  - more modeling and diagnostics for carbon data
  - extension to generalized linear mixed models