

# Estimation Systems for Rolling Samples

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## Outline

- Introduction
  - background
  - operational considerations
  - estimation system
- Weighting alternatives
  - *ad hoc* moving average
  - time series model: exponential smoothing, local linear trend
  - nonparametric regression model: local polynomial regression
- Summary
  - assessing quality of estimation systems

## Notation

- Finite population of plots:  $U = \{1, \dots, i, \dots, N\}$
- Draw sample  $s \subset U$  (sample size  $n$ ) via  $p(\cdot)$  with probabilities  $\pi_i$
- Partition  $s$  into  $T$  panels as  $s = \cup_{h=1}^T s_{t-T+h}$ :

Element	Most Recent Observation
$i \in s_{t-T+1}$	$y_{i,t-T+1}$
$\vdots$	$\vdots$
$i \in s_{t-T+h}$	$y_{i,t-T+h}$
$\vdots$	$\vdots$
$i \in s_{t-T+T}$	$y_{i,t}$

- Suppose  $\Pr[i \in s_{t-T+h} \mid s] = 1/T$
- Population mean at time  $t$ :

$$\theta_t = N^{-1} \sum_{i \in U} y_{i,t}$$

## Operational Considerations

- Not interested in *single* study variable
  - many study variables on survey instrument
  - thousands of point estimates in core tables
  - unlimited number of possible derived variables (transformations, sub-domains, lagged differences, cross-products, etc.)
- Want internal consistency
  - additivity of hierarchical subpopulations; tables should “add up”
- Want to avoid detailed modeling operations
  - too many variables
  - too few resources: time, money, personnel
  - potential for controversy among end users

## Estimation System

- Want straightforward estimates of
  - status
  - one-year change, two-year change, ..., five-year change, ...
  - standard errors
- Estimation system: data structure plus estimation routines
- Should almost run by itself
  - simple data structure
  - simple routines to get estimates from data

## Unbiased Estimation

- Could estimate  $\theta_t$  using direct estimate

$$\hat{\theta}_t = N^{-1} \sum_{i \in s_t} \frac{y_{i,t}}{\pi_i(1/T)}$$

- Design unbiased: think two-phase sample  $s_t \subset s \subset U$

$$\begin{aligned} \mathbb{E}_p [\hat{\theta}_t] &= \mathbb{E}_p \left[ \mathbb{E} \left[ N^{-1} \sum_{i \in s_t} \frac{y_{i,t}}{\pi_i(1/T)} \mid s \right] \right] \\ &= \mathbb{E}_p \left[ N^{-1} \sum_{i \in s} \frac{y_{i,t}}{\pi_i} \right] \\ &= \theta_t \end{aligned}$$

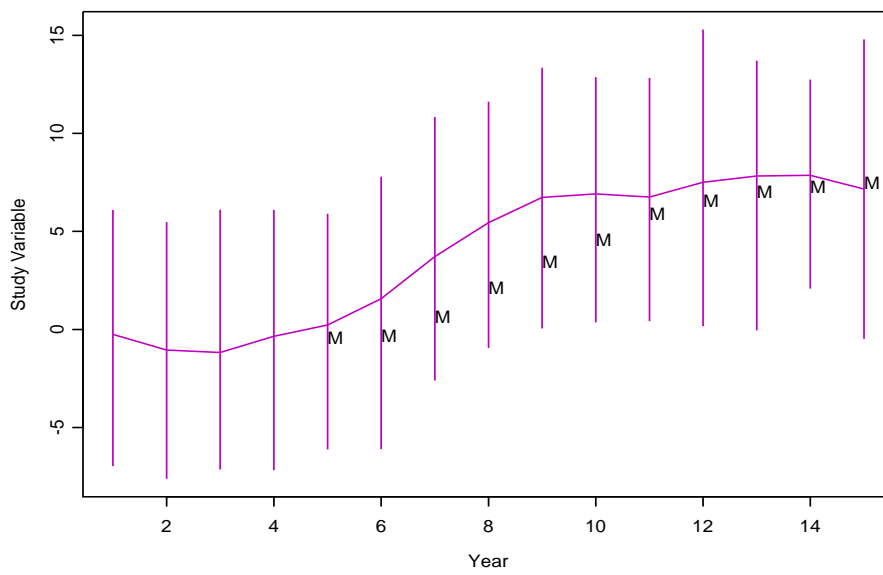
- Variance may be large due to small sample size

## Trading Bias Against Variance

- Small area estimation problem
- Use observations from past years
  - borrows strength across time
  - increases sample size
  - reduces variance
- Increases bias unless population is unchanged over time
  - time delay bias

## Time Delay Bias

- Example: simple moving average of design-based estimates
  - two-year delay



## A Class of Estimation Systems

- Questions: *importance of operational considerations? simplicity? internal consistency? statistical efficiency? reliance on models? how much past data? credibility? status versus change? revisions?*
- Start with candidate class of estimation systems
- Candidate class: build weighted data set of  $n$  records
  - one record for each plot
  - includes one or more weights for each record
  - includes most recent observation of each study variable
- Weighted data set summarizes most recent  $T$  years of data
  - may depend on past by defining new study variables:

$$y_{i,t-T+h}^* = y_{i,t-2T+h}$$

## Weighting

- Construct  $n$  weights  $\{\omega_{i,t-T+h}\}$  for  $i \in s_{t-T+h}$ ,  $h = 1, \dots, T$ 
  - reflect design properties
  - incorporate auxiliary information
  - apply to any study variable
- For any study variable  $y$ , estimate  $\theta_t = N^{-1} \sum_{i \in U} y_{i,t}$  via

$$\hat{\theta}_t = \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \omega_{i,t-T+h} y_{i,t-T+h}$$

- Internally consistent:

$$\begin{aligned} \hat{\theta}_{y+z,t} &= \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \omega_{i,s_{t-T+h}} (y_{i,t-T+h} + z_{i,t-T+h}) \\ &= \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \omega_{i,s_{t-T+h}} y_{i,t-T+h} + \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \omega_{i,s_{t-T+h}} z_{i,t-T+h} \\ &= \hat{\theta}_{y,t} + \hat{\theta}_{z,t} \end{aligned}$$

## Data Structure

- Tabulation data set:  $n$  records with one weight per record

$$\begin{bmatrix} [\omega_{i,s_{t-T+1}} & y_{i,t-T+1}^{(1)} & \cdots & y_{i,t-T+1}^{(J)}]_{i \in s_{t-T+1}} \\ \vdots & \vdots & & \vdots \\ [\omega_{i,s_{t-1}} & y_{i,t-1}^{(1)} & \cdots & y_{i,t-1}^{(J)}]_{i \in s_{t-1}} \\ [\omega_{i,s_t} & y_{i,t}^{(1)} & \cdots & y_{i,t}^{(J)}]_{i \in s_t} \end{bmatrix}$$

- Sum of weight times study variable is status estimate

## Weight Generation?

- Unmotivated, *ad hoc*
  - simple moving average
- Motivated by time series model
  - exponentially weighted moving average
  - local linear trend
  - simple moving average
- Motivated by nonparametric regression model
  - local polynomial regression
  - simple moving average

## A Simple Time Series Model

- Time series: sequence of design-based estimates,  $\hat{\theta}_t$
- Model: random walk plus noise (ARIMA(0,1,1))

$$\begin{aligned}\hat{\theta}_t &= \theta_t + \epsilon_t, & \{\epsilon_t\} \text{ iid } N(0, \sigma^2) \\ \theta_t &= \theta_{t-1} + \eta_t, & \{\eta_t\} \text{ iid } N(0, \sigma_\eta^2)\end{aligned}$$

- $\sigma_\eta^2$  is “signal” variance;  $\sigma^2$  is sampling error variance

$$\text{signal-to-noise ratio} = \text{SNR} = \frac{\sigma_\eta^2}{\sigma^2}$$

## Exponential Smoothing

- Define first difference matrix

$$\Delta_1 = \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \end{bmatrix}$$

- Best mean square predictor of  $(\theta_{t-T+1}, \dots, \theta_t)'$  is

$$\left\{ \mathbf{I} - \Delta_1' \left( \frac{\sigma_\eta^2}{\sigma^2} \mathbf{I} + \Delta_1 \Delta_1' \right)^{-1} \Delta_1 \right\} \begin{bmatrix} \hat{\theta}_{t-T+1} \\ \vdots \\ \hat{\theta}_t \end{bmatrix}$$

## Exponential Smoothing Weights

- Last row of matrix

$$\mathbf{I} - \Delta_1' \left( \frac{\sigma_\eta^2}{\sigma^2} \mathbf{I} + \Delta_1 \Delta_1' \right)^{-1} \Delta_1$$

gives weights for estimating  $\theta_t$ :

SNR	Time				
	$t - 4$	$t - 3$	$t - 2$	$t - 1$	$t$
0.0	0.20	0.20	0.20	0.20	0.20
0.1	0.04	0.08	0.12	0.26	0.50
1.0	0.02	0.04	0.10	0.24	0.62
2.0	0.00	0.02	0.06	0.20	0.74
10.0	0.00	0.00	0.00	0.08	0.92
$\infty$	0.00	0.00	0.00	0.00	1.00

- as  $\text{SNR} \rightarrow 0$ , signal  $\rightarrow$  constant
- as  $\text{SNR} \rightarrow \infty$ , sampling error  $\rightarrow 0$
- need to choose SNR to get estimation system

## Concerns about Exponential Smoothing

- Restrictiveness of model?
- Need to specify one SNR for all variables
- Implicit first difference method
  - may have trouble with curvature
- Estimation of change:
  - model implies study variable follows ARIMA(0,1,1)
  - five-year change in variable follows ARIMA(0,1,6)
- Variance estimation?
  - correlation induced by averaging
- Incorporation of auxiliary information?

## Local Linear Trend Model

- Time series: sequence of design-based estimates,  $\hat{\theta}_t$
- Model: local linear trend plus noise (ARIMA(0,2,2))

$$\begin{aligned}\hat{\theta}_t &= \theta_t + \epsilon_t, & \{\epsilon_t\} &\text{ iid } N(0, \sigma^2) \\ \theta_t &= \theta_{t-1} + \beta_{t-1} + \eta_t, & \{\eta_t\} &\text{ iid } N(0, \sigma_\eta^2) \\ \beta_t &= \beta_{t-1} + \xi_t, & \{\xi_t\} &\text{ iid } N(0, \sigma_\xi^2)\end{aligned}$$

- Define the second difference matrix

$$\mathbf{\Delta}_2 = \begin{bmatrix} 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & 1 & -2 & 1 \end{bmatrix}$$

- Weights for current status given by last row of

$$\mathbf{I} - \mathbf{\Delta}'_2 \left( \frac{\sigma_\xi^2}{\sigma^2} \mathbf{I} + \frac{\sigma_\eta^2}{\sigma^2} \mathbf{\Delta}_1 \mathbf{\Delta}'_1 + \mathbf{\Delta}_2 \mathbf{\Delta}'_2 \right)^{-1} \mathbf{\Delta}_2$$

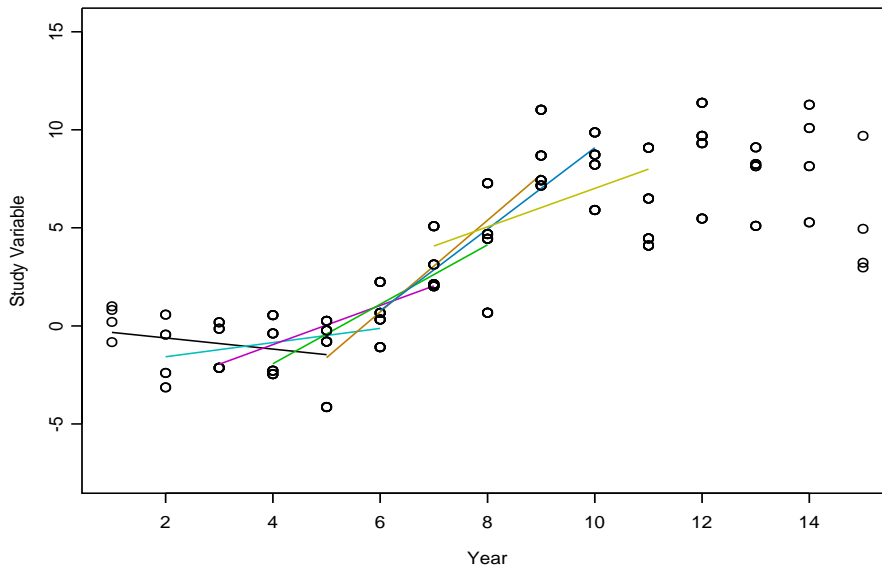
– need to choose two variance ratios to get estimation system

## Concerns

- Restrictiveness of model?
- Need to specify one pair of variance ratios for all variables
- Possibility of negative weights
- Estimation of change:
  - model implies study variable follows  $ARIMA(0,2,2)$
  - five-year change in variable follows  $ARIMA(0,2,7)$
- Variance estimation?
  - correlation induced by averaging
- Incorporation of auxiliary information?

# Nonparametric Regression

- Signal can be modeled as:
  - positively autocorrelated stochastic process: time series
  - smooth deterministic function of time: nonparametric regression
- Local linear regression:



## Local Polynomial Regression

- Define the  $n \times (q + 1)$  matrix

$$\mathbf{X}_t = \left[ \left[ 1 \quad h \quad \dots \quad h^q \right]_{i \in s_{t-T+h}} \right]_{h=1}^T,$$

the  $n \times n$  matrix

$$\mathbf{W}_t = \text{blockdiag} \left\{ \text{diag} \left\{ \frac{1}{\pi_i} \right\}_{i \in s_{t-T+h}} \right\}_{h=1}^T,$$

and the  $n \times 1$  vector

$$\mathbf{y}_t = \left[ \left[ y_{i,t-T+h} \right]_{i \in s_{t-T+h}} \right]_{h=1}^T.$$

- Obtain local  $q$ th order polynomial fit via weighted least squares:

$$\begin{aligned} \hat{m}(t - T + k) &= \left[ 1 \quad k \quad \dots \quad k^q \right] (\mathbf{X}'_t \mathbf{W}_t \mathbf{X}_t)^{-1} \mathbf{X}'_t \mathbf{W}_t \mathbf{y}_t \\ &= \left[ 1 \quad k \quad \dots \quad k^q \right] (\mathbf{X}'_t \mathbf{W}_t \mathbf{X}_t)^{-1} \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \begin{bmatrix} 1 \\ h \\ \vdots \\ h^q \end{bmatrix} \frac{1}{\pi_i} y_{i,t-T+h} \end{aligned}$$

## Two-Phase Regression Estimator

- Two-phase sampling with  $s_t \subset s \subset U$ :
  - predict  $y_{i,t}$  by  $\hat{y}_{i,t}^{(U)}$  for  $i \in U$
  - predict  $y_{i,t}$  by  $\hat{y}_{i,t}^{(s)}$  for  $i \in s$
  - predict  $y_{i,t}$  by  $y_{i,t}$  for  $i \in s_t$
- Two-phase regression estimator (Särndal, Swensson, and Wretman, 1992, Ch. 9):

$$\sum_{i \in U} \frac{\hat{y}_{i,t}^{(U)}}{N} + \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \frac{\hat{y}_{i,t}^{(s)} - \hat{y}_{i,t}^{(U)}}{N\pi_i} + \sum_{i \in s_t} \frac{y_{i,t} - \hat{y}_{i,t}^{(s)}}{N\pi_i(1/T)}$$

- approximately design-unbiased
- good MSE properties if predictions are good

## Local Polynomial Regression Predictions

- Plot-level model:

$$y_{i,t} = m(t) + \delta_i + \epsilon_{it}$$

- $m(t)$  is a smooth function of time
- $\{\delta_i\}$  iid  $(0, \sigma_\delta^2)$
- $\{\epsilon_{it}\}$  zero-mean stochastic processes, independent across  $i$

- Predictions:

- predict  $y_{i,t}$  by  $\hat{y}_{i,t}^{(U)} = \hat{m}(t)$  for  $i \in U$
- predict  $y_{i,t}$  by  $\hat{y}_{i,t}^{(s)} = \hat{m}(t) + y_{i,t-T+h} - \hat{m}(t - T + h)$  for  $i \in s_{t-T+h}$
- predict  $y_{i,t}$  by  $y_{i,t}$  for  $i \in s_t$

- Note:

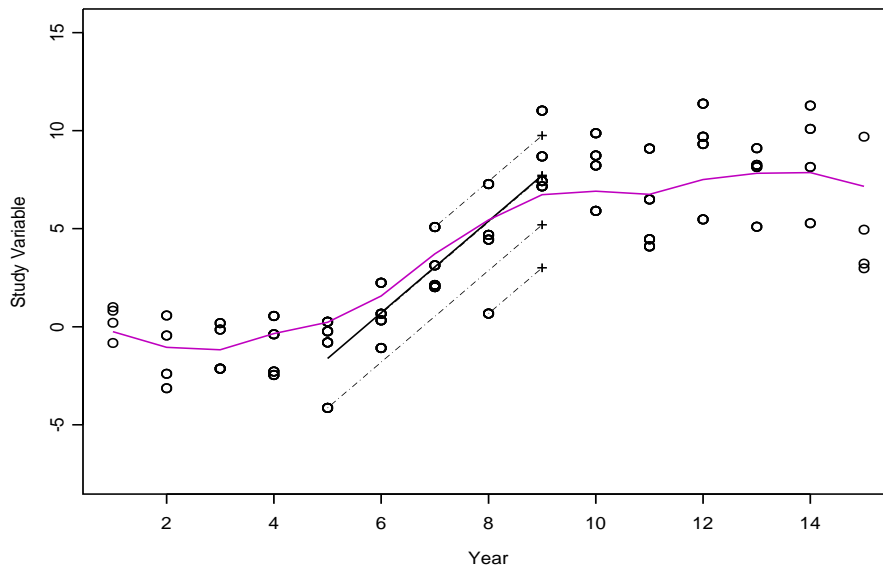
$$\sum_{h=1}^T \sum_{i \in s_{t-T+h}} \frac{\hat{y}_{i,t}^{(s)} - \hat{y}_{i,t}^{(U)}}{N\pi_i} = 0$$

and

$$\sum_{i \in s_t} \frac{y_{i,t} - \hat{y}_{i,t}^{(s)}}{N\pi_i(1/T)} = 0$$

## Graphical Interpretation

- Predictions:  $\hat{y}_{i,t}^{(s)} = \hat{m}(t) + y_{i,t-T+h} - \hat{m}(t-T+h)$



## Local Polynomial Regression Estimator

- Local polynomial regression estimator of  $\theta_t$  is given by

$$\begin{aligned}\tilde{\theta}_t &= \sum_{i \in U} \frac{\hat{y}_{i,t}^{(U)}}{N} + \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \frac{\hat{y}_{i,t}^{(s)} - \hat{y}_{i,t}^{(U)}}{N\pi_i} + \sum_{i \in s_t} \frac{y_{i,t} - \hat{y}_{i,t}^{(s)}}{N\pi_i(1/T)} \\ &= \hat{m}(t)\end{aligned}$$

- approximately design-unbiased
- good MSE properties if predictions are good

## Plot Weights for Local Linear Regression

- Weighted form of local linear regression estimator:

$$\begin{aligned}\tilde{\theta}_t &= \hat{m}(t) = \begin{bmatrix} 1 & T \end{bmatrix} (\mathbf{X}'_t \mathbf{W}_t \mathbf{X}_t)^{-1} \mathbf{X}'_t \mathbf{W}_t \mathbf{y}_t \\ &= \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \begin{bmatrix} 1 & T \end{bmatrix} \left\{ (\mathbf{X}'_t \mathbf{W}_t \mathbf{X}_t)^{-1} \begin{bmatrix} 1 \\ h \end{bmatrix} \frac{1}{\pi_i} \right\} y_{i,t-T+h} \\ &= \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \begin{bmatrix} 1 & T \end{bmatrix} \boldsymbol{\omega}_{i,t-T+h} y_{i,t-T+h} \\ &= \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \omega_{i,t-T+h} y_{i,t-T+h}\end{aligned}$$

- easy to compute
- can be rearranged to get moving average of design-based estimators

## Local Polynomial Regression: Moving Average Weights

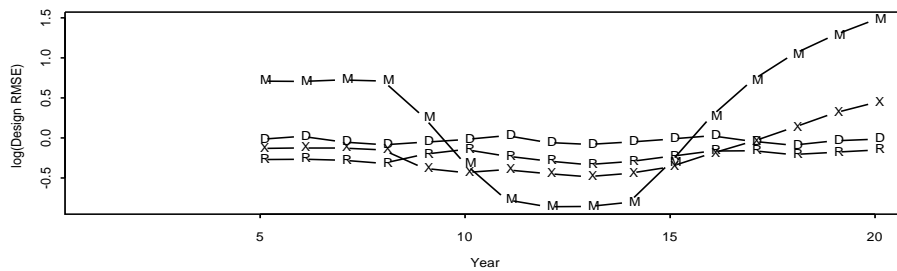
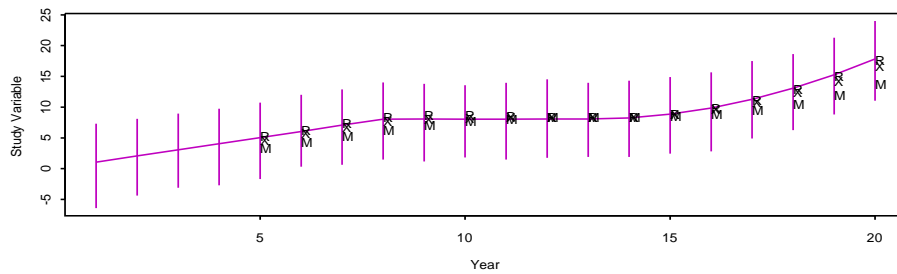
- Moving average weights for  $T = 5$

Polynomial Order	Time				
$q$	$t - 4$	$t - 3$	$t - 2$	$t - 1$	$t$
0	0.200	0.200	0.200	0.200	0.200
1	-0.200	0.000	0.200	0.400	0.600
2	0.086	-0.143	-0.086	0.257	0.886
3	-0.014	0.057	-0.086	0.057	0.986
4	0.000	0.000	0.000	0.000	1.000

- fitting of local constant equivalent to simple moving average
- otherwise, more weight on current conditions
- negative weights imply possibility of negative estimates

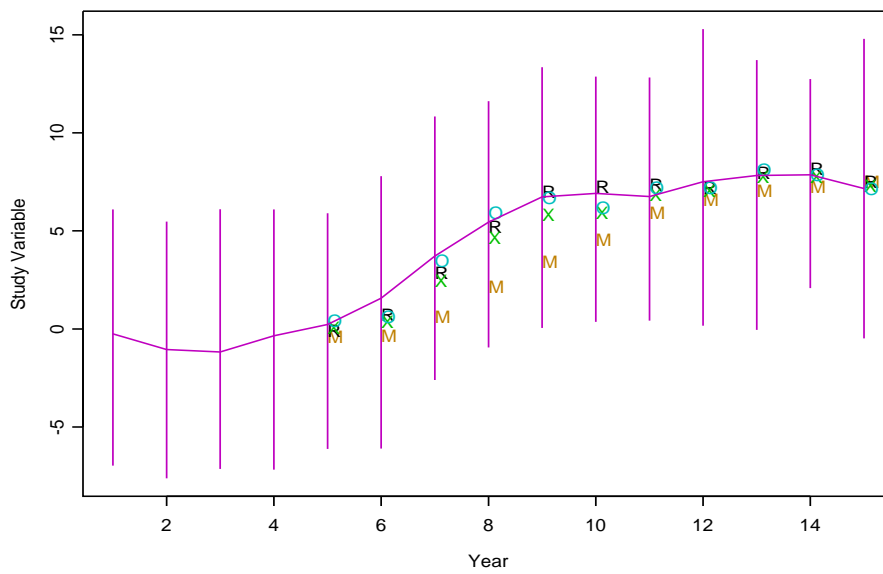
## Simulation Results

- Estimators in 100 simple random samples:
  - simple moving average (M),
  - local linear regression (R),
  - exponential smoothing with SNR= 1 (X)



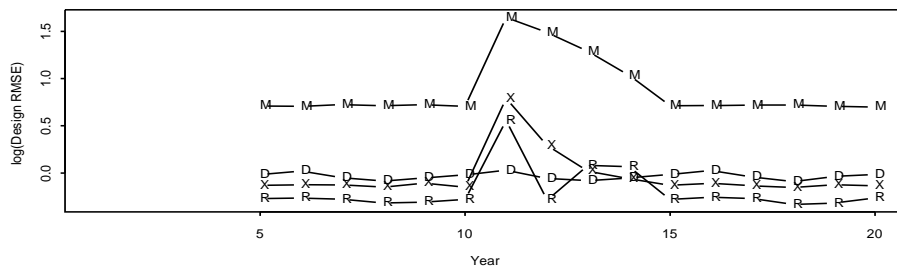
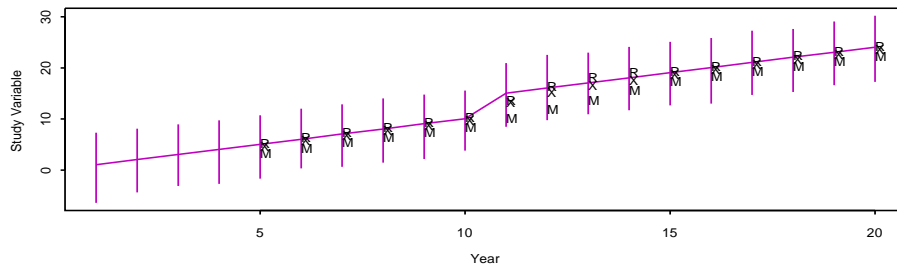
## Example: Stochastic Trend

- Stochastic trend generated by local linear trend model
  - optimal mean squared error predictor (O)



# Example: Discontinuous Trend

- Line with jump:



## Variance Estimation

- Estimator of design variance based on Taylor series linearization:

$$\begin{bmatrix} 1 & T \end{bmatrix} (\mathbf{X}'_t \mathbf{W}_t \mathbf{X}_t)^{-1} \mathbf{V}_t (\mathbf{X}'_t \mathbf{W}_t \mathbf{X}_t)^{-1} \begin{bmatrix} 1 \\ T \end{bmatrix}$$

where

$$\mathbf{V}_t = \left[ \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \sum_{k=1}^T \sum_{j \in s_{t-T+k}} \frac{\pi_{ij} - \pi_i \pi_j}{\pi_{ij}} \frac{h^f e_{i,t-T+h} k^g e_{j,t-T+k}}{\pi_i \pi_j} \right]_{f,g=0}^1$$

and

$$e_{i,t-T+h} = y_{i,t-T+h} - \hat{m}(t - T + h)$$

– simplifies for designs like simple random sampling

## Estimators of Lagged Status

- Estimators of lagged status: at time  $t$ , two-phase regression estimator of  $\theta_{t-T+k}$  ( $k = 1, \dots, T$ ) is:

$$\begin{aligned}\tilde{\theta}_{t-T+k|t} &= \hat{m}(t - T + k) \\ &= \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \begin{bmatrix} 1 & k \end{bmatrix} \boldsymbol{\omega}_{i,t-T+h} y_{i,t-T+h}\end{aligned}$$

- might be interesting to study properties of revisions; for example:

$$\tilde{\theta}_{t-T+k|t} - \tilde{\theta}_{t-T+k|t-1}$$

## Estimators of Change

- Estimator of  $k$ -year change ( $k = 1, \dots, T - 1$ ):

$$\tilde{\theta}_t - \tilde{\theta}_{t-T+k|t} = \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \begin{bmatrix} 0 & T - k \end{bmatrix} \boldsymbol{\omega}_{i,t-T+h} y_{i,t-T+h}$$

– no direct measurements of  $k$ -year change for  $k < T$

- Estimator of  $T$ -year change:

– define direct measurements of  $T$ -year change

$$y_{i,t-T+h}^* = y_{i,t-T+h} - y_{i,t-2T+h}$$

– change estimator is

$$\sum_{h=1}^T \sum_{i \in s_{t-T+h}} \begin{bmatrix} 1 & T \end{bmatrix} \boldsymbol{\omega}_{i,t-T+h} y_{i,t-T+h}^*$$

– nonparametric model still holds for  $T$ -year difference

$$m^*(t) = m(t) - m(t - T)$$

## Data Structure

- Tabulation data set:  $n$  records with two weights per record

$$\begin{bmatrix} [\omega'_{i,s_{t-T+1}} & y_{i,t-T+1}^{(1)} & \cdots & y_{i,t-T+1}^{(J)}]_{i \in s_{t-T+1}} \\ \vdots & \vdots & & \vdots \\ [\omega'_{i,s_{t-1}} & y_{i,t-1}^{(1)} & \cdots & y_{i,t-1}^{(J)}]_{i \in s_{t-1}} \\ [\omega'_{i,s_t} & y_{i,t}^{(1)} & \cdots & y_{i,t}^{(J)}]_{i \in s_t} \end{bmatrix}$$

- Sum of study variable times weight vector times
  - $(1, T)'$  is current status estimate
  - $(1, k)'$  is lagged status estimate
  - $(0, T - k)'$  is  $k$ -year change estimate

## Incorporating Auxiliary Information

- Have  $r$  regressors  $\mathbf{x}_{i,t-T+h}$  for  $i \in s_{t-T+h}$  and know  $\sum_{i \in U} \mathbf{x}_{i,t}$ 
  - GIS coverage
  - imagery

- Plot-level model:

$$y_{i,t} = m(t) + \mathbf{x}'_{it} \boldsymbol{\gamma} + \delta_i + \epsilon_{it}$$

- $m(t)$  is a smooth function of time
  - $\boldsymbol{\gamma}$  is a vector of constants
  - $\{\delta_i\}$  iid  $(0, \sigma_\delta^2)$
  - $\{\epsilon_{it}\}$  zero-mean stochastic processes
- Define the  $n \times (2 + r)$  matrix

$$\mathbf{X}_t = \left[ \left[ \begin{array}{ccc} 1 & h & \mathbf{x}'_{i,t-T+h} \end{array} \right]_{i \in s_{t-T+h}} \right]_{h=1}^T$$

- Local regression fit given by:

$$(\mathbf{X}'_t \mathbf{W}_t \mathbf{X}_t)^{-1} \mathbf{X}'_t \mathbf{W}_t \mathbf{y}_t$$

## Predictions Using Auxiliary Information

- Model:

$$y_{i,t} = m(t) + \mathbf{x}'_{it}\boldsymbol{\gamma} + \delta_i + \epsilon_{it}$$

- Predictions:

– predict  $y_{i,t}$  by  $\hat{y}_{i,t}^{(U)} = \hat{m}(t) + \mathbf{x}'_{it}\hat{\boldsymbol{\gamma}}$  for  $i \in U$

– predict  $y_{i,t}$  by  $\hat{y}_{i,t}^{(s)} = \hat{m}(t) + \mathbf{x}'_{it}\hat{\boldsymbol{\gamma}} + y_{i,t-T+h} - \hat{m}(t-T+h) - \mathbf{x}'_{it}\hat{\boldsymbol{\gamma}}$   
for  $i \in s_{t-T+h}$

– predict  $y_{i,t}$  by  $y_{i,t}$  for  $i \in s_t$

- Note:

$$\sum_{h=1}^T \sum_{i \in s_{t-T+h}} \frac{\hat{y}_{i,t}^{(s)} - \hat{y}_{i,t}^{(U)}}{N\pi_i} = 0$$

and

$$\sum_{i \in s_t} \frac{y_{i,t} - \hat{y}_{i,t}^{(s)}}{N\pi_i(1/T)} = 0$$

## Two-Phase Regression Estimator Using Auxiliary Information

- Two-phase regression estimator is then

$$\begin{aligned}\tilde{\theta}_t &= \sum_{i \in U} \frac{\hat{m}(t) + \mathbf{x}'_{it} \hat{\boldsymbol{\gamma}}}{N} \\ &= \left[ 1 \quad T \quad N^{-1} \sum_{i \in U} \mathbf{x}'_{it} \right] (\mathbf{X}'_t \mathbf{W}_t \mathbf{X}_t)^{-1} \mathbf{X}'_t \mathbf{W}_t \mathbf{y}_t \\ &= \sum_{h=1}^T \sum_{i \in s_{t-T+h}} \omega_{i,t-T+h} y_{i,t-T+h}\end{aligned}$$

– weights have *calibration* property:

$$\sum_{h=1}^T \sum_{i \in s_{t-T+h}} \omega_{i,t-T+h} \mathbf{x}_{i,t-T+h} = N^{-1} \sum_{i \in U} \mathbf{x}_{i,t}$$

- May be trouble with timing, availability of auxiliary information
  - literature on “benchmarking” of survey data may be useful

## Further Work on Regression Approach

- Negative estimates?
  - negative weight on most distant past observation
  - negative estimate indicates trouble (unusually small subpopulation, unusual trend)
  - *ad hoc* fix via composite estimation:

$$(1 - k)\tilde{\theta}_t + k\hat{\theta}_t$$

- More complex panel structure?
  - can always reduce to case considered here
- Start up?
- Benchmarking? (irregularly timed remote sensing data)

## Summary

- Estimation system: data structure plus estimation routines
  - must handle huge number of study variables
  - should yield internally consistent estimates
  - should have simple data structure, estimation routines
  - should borrow strength across time
  - should be able to incorporate auxiliary information
  - should not be overly dependent on modeling assumptions
  - should have reasonable efficiency for status and change estimates
  - (in particular, should be better than direct estimates)