Small Area Estimation for Natural Resource Surveys

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Outline

- Primer on small area estimation
  - direct and indirect estimation
  - synthetic and composite estimation
  - borrowing strength and shrinkage
- Small area estimation examples
  - semi-parametric small area estimation
  - constrained estimation for ensembles

Domains

- $Domain =$ subpopulation of interest in a survey
  - geographic domains = areas (ecoregion, state, county, HUC)
- Major domains:
  - sufficient sample size allocated at the design stage
  - standard survey estimation procedures yields estimates of adequate precision
  - may be addressed by regulation, such as CWA 305(b)

Major Domains: Use Direct Estimation

G GENERAL AUDIENCES
All Ages Admitted

- Direct estimators:
  - use data only from the study units in the domain and time period of interest
  - include standard weighted survey estimators
  - good design properties: unbiased estimator and valid confidence intervals *without any statistical model!*
- Direct estimation is not reliable if sample size is extremely small

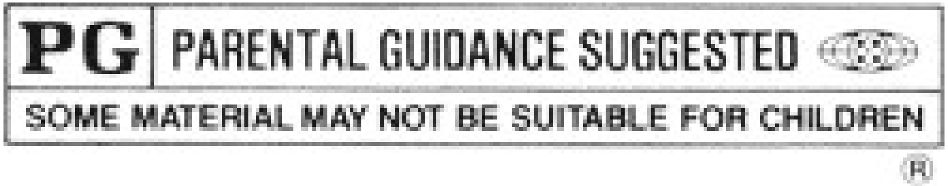
Small Domains: Direct Estimates Not Reliable

- Small domains/Small areas
  - sample size is small and may be zero in some domains
  - model-based inference is necessary to yield estimates of adequate precision
  - (definition depends on sampling resources and precision requirements)
- Typical map is covered with small areas

Indirect Estimation: Borrowing Strength

- Indirect estimators:
  - use data from outside the domain and/or time period of interest
  - (time indirect, domain indirect, domain and time indirect)
  - explicitly use statistical model to “borrow strength” across time or space
  - include various small area estimators

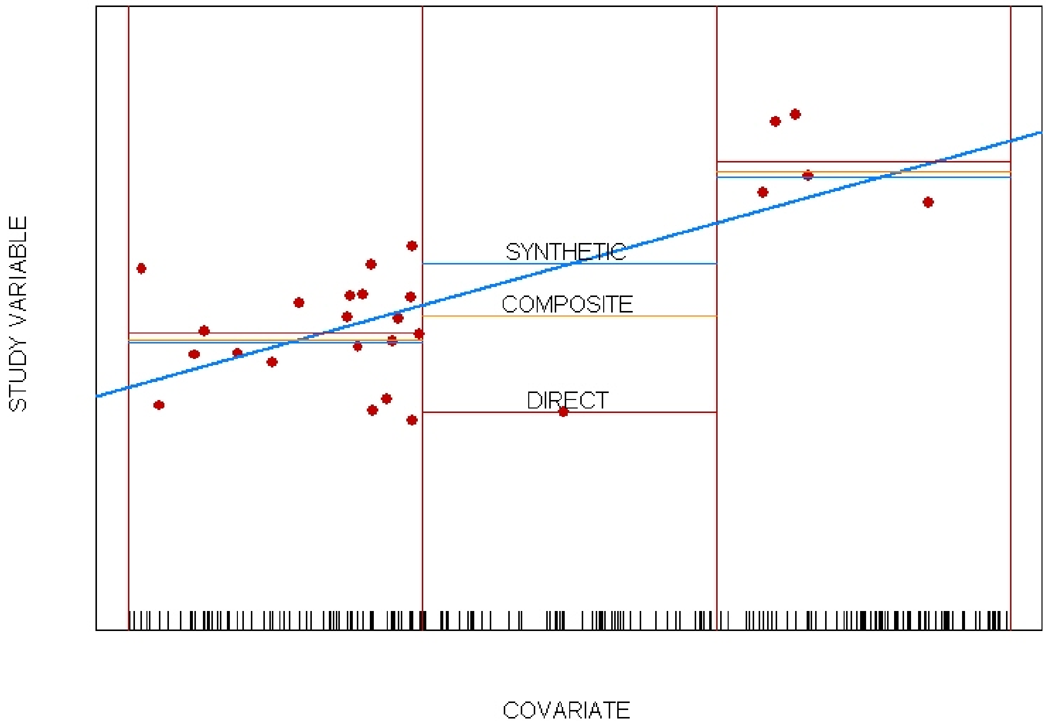
Indirect Estimation: Synthetic Estimator



- Have: response variables for sample, covariates for entire landscape
- Fit "global" model relating response variable to covariates
- Predict response variable at unobserved locations using available covariates and fitted model
  - works even if no samples in the area
  - may be poor if model is incorrectly specified

Direct, Synthetic and Composite Estimators

- One covariate, three small areas



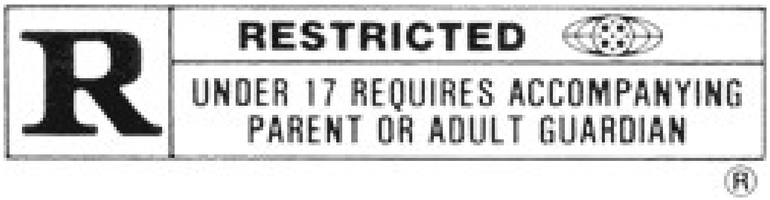
Shrinkage in the Composite Estimator

- Direct is moved toward synthetic to get composite estimator
  - equivalently, small-area specific effect "shrinks toward zero"
- Much of small area estimation involves choosing the shrinkage factor
- *Ad hoc* composite estimator

$$\text{composite} = w_h(\text{direct}) + (1 - w_h)(\text{synthetic})$$

  - still rated $\mathbf{PG}$

Formal Composite Estimation



- $w_h =$ function of parameters from a fitted mixed model
- Mature audiences only:
  - good auxiliary information
  - correctly-specified global regression structure
  - correctly-specified local correlation structure
  - (may require violence or coarse language)
  - sexy models and methods: EBLUP/EB, HB

Small Area Models

- Model for direct estimates:

$$\begin{aligned}\hat{\theta}_h &= \text{direct estimate for small area } h \\ &= \theta_h + e_h \\ &= \text{truth+sampling error} \\ \theta_h &= \mathbf{x}_h^T \boldsymbol{\beta} + \omega_h, \quad [\omega_h] \sim (\mathbf{0}, \Gamma) \\ &= \text{regression + area-specific deviation}\end{aligned}$$

- Two ways to borrow strength:
  - globally, through regression fitted to all data
  - locally, through spatially (or temporally) correlated random effects

Borrowing Strength

- Only global, through regression fitted to all data

$$\mathbf{x}_h^T \hat{\boldsymbol{\beta}} = \mathbf{PG}\text{-rated synthetic estimator}$$

- Both global and local, allowing spatially (or temporally) correlated random effects

$$\mathbf{x}_h^T \hat{\boldsymbol{\beta}} + \hat{\omega}_h = \mathbf{R}\text{-rated composite estimator}$$

Fitting Small Area Models

- Model:

$$\hat{\theta}_h = \theta_h + e_h$$
$$\theta_h = \mathbf{x}_h^T \boldsymbol{\beta} + \omega_h, \quad [\omega_h] \sim (\mathbf{0}, \Gamma)$$

- Empirical BLUP/ Empirical Bayes: Compute $\hat{\boldsymbol{\beta}}$ and $\hat{\Gamma}$ and plug in to get

$$\tilde{\theta}_h = \mathbf{x}_h^T \hat{\boldsymbol{\beta}} + \hat{\omega}_h$$

- Hierarchical Bayes (HB): Compute $\boldsymbol{\beta} \,|\, \text{data}$ and $\Gamma \,|\, \text{data}$ and plug in to get

$$\theta_h \,|\, \text{data}$$

Comparison of Estimation Methods

- Empirical BLUP/ Empirical Bayes:
  - relatively straightforward computation
  - can use SAS `proc mixed` or S-Plus function `lme()`
  - does not fully account for uncertainty due to unknown variance components
- Hierarchical Bayes:
  - entire posterior distribution, not just point estimates
  - full accounting for uncertainty (assuming correct model specification)
  - computation is typically much more involved, though sometimes can be done in `winbugs`

Numerical Implementation of Hierarchical Bayes

- Markov chain Monte Carlo (MCMC): often necessary to approximate posterior distribution of unknowns given data
- Idea: any distribution can be studied provided we can simulate from it
  - iid draws from distribution would be ideal
  - dependent, identically distributed draws would be fine if dependence is not too strong (ergodic theorem)
  - dependent, nearly identically distributed draws might be OK

Markov Chain Monte Carlo (MCMC)

- MCMC generates Markov chain with invariant distribution equal to posterior distribution of interest
  - not independent due to Markov structure
  - not identically distributed except asymptotically, due to initialization problem
  - assessing convergence is critical
- MCMC recipes for constructing suitable Markov chains include
  - Gibbs sampler
  - Metropolis-Hastings algorithm

Cautions on Small Area Estimation

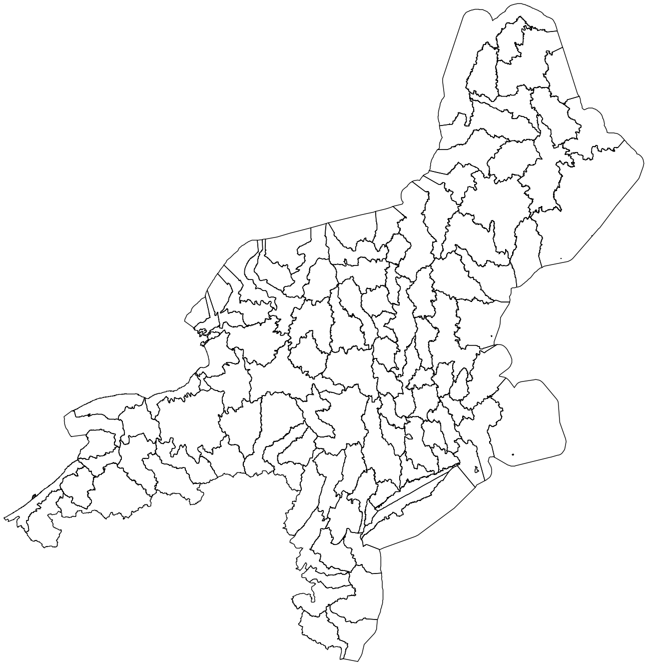
- Be wary of unreasonable expectations
- Global regression:
  - covariates from remote sensing may have poor explanatory power
  - particularly true for aquatic responses
- Local scale:
  - spatial correlations may be weak at scale of probability sample
  - particularly true once useful covariates taken into account
  - sampling may not be sufficiently dense for good estimates of spatial structure

Two Small Area Estimation Problems

- Acid Neutralizing Capacity (ANC)
  - surface waters are acidic if ANC < 0
  - supply of acids from atmospheric deposition and watershed processes exceeds buffering capacity
- ANC level: Semiparametric small area estimation
  - HUCs in Northeast
- ANC trend: Constrained ensemble estimates
  - HUCs in mid-Atlantic highlands

Semiparametric Small Area Estimation of ANC Level

- Joint work with J. Opsomer, G. Ranalli, G. Claeskens, G. Kauermann
- 557 observations over 113 HUCs



HUCs as Small Areas

- Few sample observations available in most HUCs
  - Average sample size/HUC: 4.9
  - 64 HUCs contain less than 5 observations
  - 27 out of 113 HUCs contain no sample observations
- Site-specific covariates: lake location and elevation
  - not much available for global regression
  - local structure: can use correlation or flexible trend

Semiparametric Small Area Model

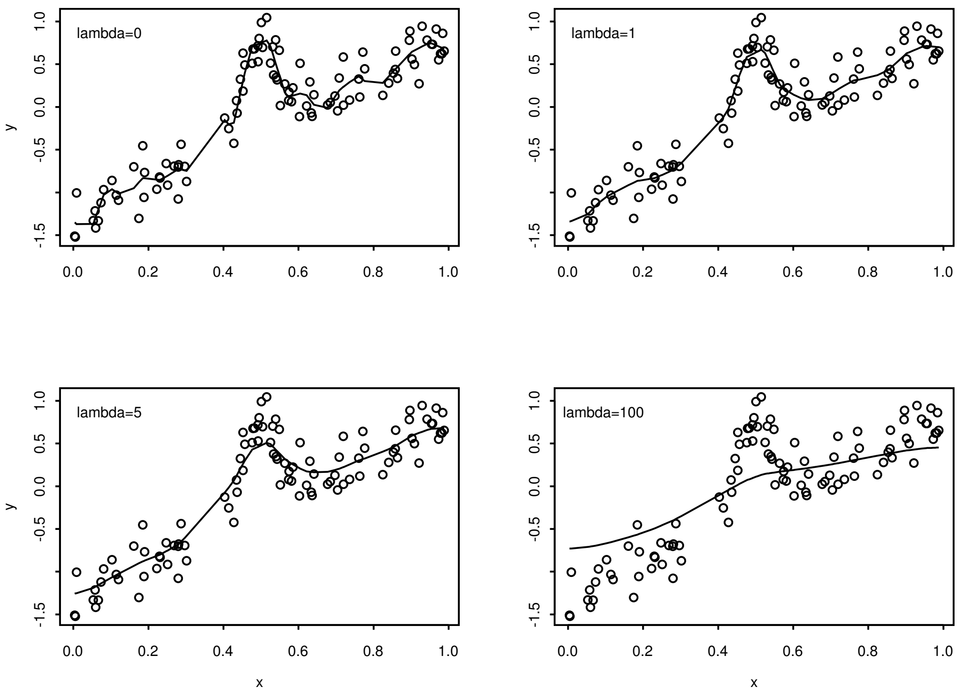
- Replace linear function of covariates by more general model:

$$
\begin{aligned}
\text{direct} &= \text{truth}+\text{sampling error} \\
\text{truth} &= m(\mathbf{x}_h; \boldsymbol{\gamma}) + \omega_h \\
&= \text{semiparametric regression} + \text{area-specific deviation} \\
&= \mathbf{x}_h^T \boldsymbol{\beta} + \mathbf{z}_h^T \boldsymbol{\alpha} + \omega_h
\end{aligned}
$$

- Semiparametric regression expressed as mixed linear model
  - penalized splines (P-splines)
  - thin plate splines
  - kriging
- EBLUP easily handled with standard software (SAS, SPlus)

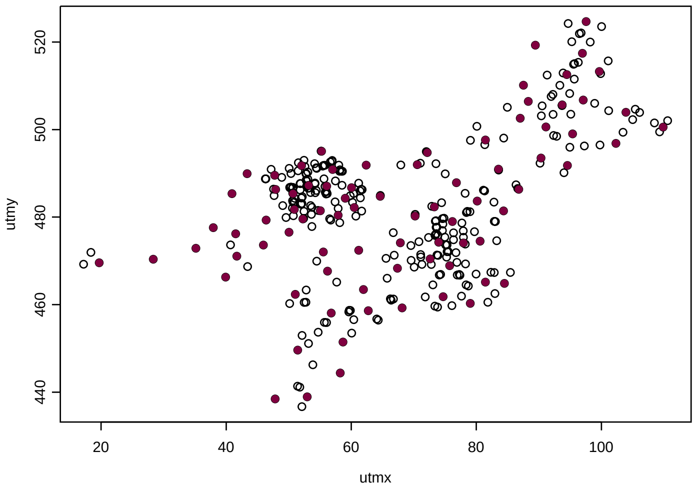
Fitting by Penalized Splines Regression

- Allow slope changes at each of many knots
  - penalize excessive slope changes via λ



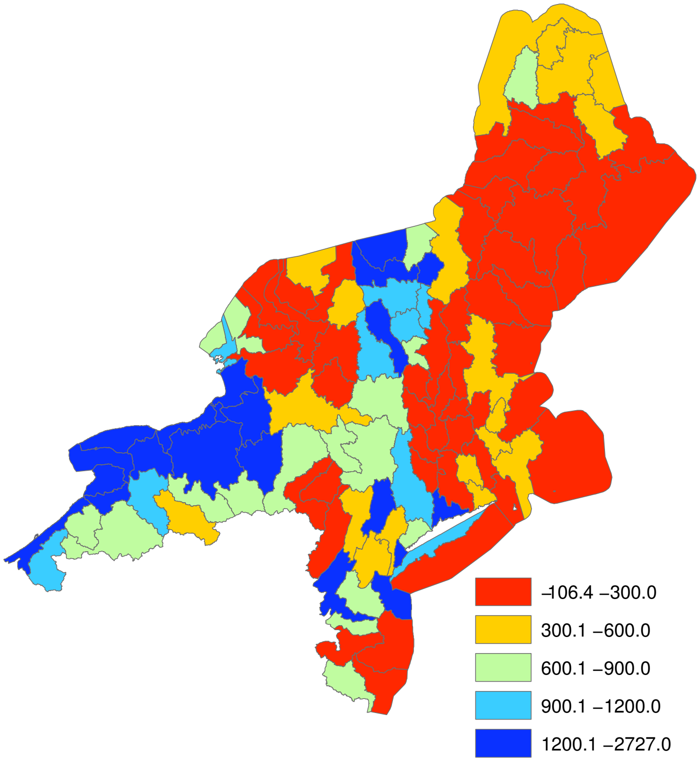
Spatial Smoothing Using P-Splines

- NE Lakes data require bivariate (spatial) smoothing $\approx$ thin-plate spline (Ruppert *et al.* 2003)
- Knot selection: space-filling algorithm



NE Lakes HUC Predictions

- Correlation between ANC and model prediction: 0.96



Constrained Bayes Estimation for ANC Trend

- Mark Delorey (PhD student)
  - "Precision Monitoring Approaches for Research and Management Applications" session
- Interested in estimating **individual** HUC-specific slopes
- Also interested in **ensemble**:
  spatially-indexed true values: $\{\tau_h\}_{h=1}^{m}$
  spatially-indexed estimates: $\{\tau_h^{\text{est}}\}_{h=1}^{m}$
  - **subgroup analysis**: what proportion of HUC's have ANC decreasing over time?
- Hierarchical Bayes "overshrinks" in this context

Small Area Estimation Summary

- $\mathbf{G}$-rated direct estimates: no shrinkage
- Indirect estimates: $\mathbf{PG}$ or $\mathbf{R}$
  - need good covariates and/or useful correlations
  - rare in aquatic resources
- Shrinkage:
  - none = direct: $\mathbf{G}$-rated
  - total = synthetic: $\mathbf{PG}$-rated
  - ad hoc composite: $\mathbf{PG}$-rated
  - formal composite: $\mathbf{R}$-rated
- Two examples: semiparametric and constrained