Keeling and Gilligan papers: Metapopulation dynamics of bubonic plague.

Deterministic model: 5 differential equations describing dynamics of rats & fleas.

Variables:

rats

S_r = number of susceptible rats

I_r = number of infectious rats

R_r = number of resistant rats

$T_r = S_r + I_r + R_r$ = total # of rats

fleas

N = average # of fleas on a rat
- flea index

F = total # of infected fleas searching for a host

humans

λ_H = potential force of infection to humans

Parameters:

	Value
r_R - rat productive rate	5
p - prob of inherited resistance	.975
K_R - rat carrying capacity	2500
d_r - natural rat death rate	.2
β_R - contact rate with fleas	4.7
m_R - 1/(infectious period)	20
g_R - prob of recovery	.02
a - flea searching efficiency	.004
r_F - flea reproductive rate	20
d_F - natural flea death rate	10
K_F - flea carrying capacity on rat	6.57

Equations

$$* \quad \frac{dS_R}{dt} = r_R \overset{S}{\cancel{R}}_R (1 - T_R/K_R) + r_R R_R(1-p) - d_R S_R - \beta_R S_R F(1-e^{-aT_R})/T_R$$

Notes

- $r_R R_R (1 - T_R/K_R)$ logistic growth
- $r_R R_R(1-p)$ resistant rats giving birth to non-resistant rats
- $d_R S_R$ natural death
- $\beta_R S_R F(1-e^{-aT_R})/T_R$ infection due to contact with a "free" searching flea. Here $1-e^{-aT_R}$ = prob free flea finds a host, so that $F(1-e^{-aT_R})/T_R$ is the expected number of (free fleas latching onto a rat)/rat.

$$* \quad \frac{dI_R}{dt} = \beta_R S_R F(1-\exp(-aT_R))/T_R - (d_R + m_R) I_R$$

Notes

- $d_R I_R$ – natural death
- $m_R I_R$ – removal from infected either by death due to disease or acquired resistance.

$$* \quad \frac{dR_R}{dt} = r_R R_R (p - T_R/K_R) + m_R g_R I_R - d_R R_R$$

Notes

- $r_R R_R (p - T_R/K_R)$ logistic growth
- $m_R g_R I_R$ infecteds who develop resistance
- $d_R R_R$ natural death.

* $\dfrac{dN}{dt} = r_F N(1 - N/K_F) + d_F F(1 - e^{-aT_R})/T_R$

Notes

- $r_F N(1 - N/K_F)$ logistic growth (Does this make sense?)
- $d_F F(1 - e^{-aT_R})/T_R$

$1 - e^{-aT_R}$ = prob of free flea finds a host.

So

(1) $$\frac{F(1 - e^{-aT_R})}{T_R} = \frac{\text{exp \# of free fleas finding a host}}{\text{total \# of rats}}$$

In personal communication with Jay, Keeling says the "prob of finding a new rat host must be spread across the searching time of flea (ie the time until it starves to death)." The mean time to death is approximately

$1/d_F$.

Thus we need to divide (1) by this value which gives

$$\frac{1}{1/d_F} F(1 - e^{-aT_R})/T_R.$$

* $\dfrac{dF}{dt} = (d_R + m_R(1 - g_R)) I_R N - d_F F$

Notes

- $(d_R + m_R(1 - g_R)) I_R N$ infected rat dies and releases N inflectious fleas.
- $d_F F$ natural death rate of fleas.

* $\lambda_H = F \exp(-aT_R)$ - expected number of fleas that do not find a rat host.

Stochastic Implementation

STREUDLize.

Remark: If an infected rat dies, he/she releases a random number X of fleas where X has a Poisson distribution with mean N - flea index

$(S, I, R, N, F) \longrightarrow$

	rate	interpretation
$(S+1, I, R, N, F)$		
$\vdots$		