

There are three questions, with a total of 50 points. **Please start a new page for each question and do not use both sides.**

1. [Total 15 points]

Let Y_1, \dots, Y_n denote a random sample from the probability density function,

$$f_Y(y; \theta) = \frac{\theta}{2^\theta} (2 - y)^{\theta-1}, \quad 0 < y < 2, \quad \theta > 0.$$

- (a) [5] Find the maximum likelihood estimator of θ .
- (b) [5] Find the method of moments estimator of θ .
- (c) [5] Find a sufficient estimator for θ .

2. [Total 20 points]

Let Y_1, \dots, Y_n be a random sample from

$$f_Y(y; \lambda) = \lambda^{-1} e^{-(y-1)/\lambda} \text{ for } 1 \leq y \text{ and } \lambda > 0.$$

- (a) [5] Calculate $E(Y_i)$, $E(Y_i^2)$, and $\text{Var}(Y_i)$.
- (b) [5] Find the maximum likelihood estimator of λ .
- (c) [5] Calculate the mean and variance of the maximum likelihood estimator. Is this estimator unbiased?
- (d) [5] Find the Cramer–Rao lower bound for estimating λ and compare with the variance of the maximum likelihood estimator computed in (c). What can you deduce about the ML estimator?

3. [Total 15 points]

Suppose the driving time for vans operated by an airport shuttle company from Fort Collins to DIA is $N(\mu, \sigma^2)$ distributed with $\sigma^2 = 160$ minutes.

- (a) [5] How many shuttle trips to DIA would be required to estimate μ with accuracy 5 minutes using a 95% confidence interval?
- (b) [5] A trip in the airport shuttle van is defined *to be on time* if the actual arrival time is within 5 minutes of the scheduled time. What is the probability that a single trip to DIA on the airport van is on time?
- (c) [5] If 20 out of 100 van trips to DIA are on time, construct a 95% confidence interval for the probability p that a shuttle van is on time to the airport. Does this analysis support the probability calculated in (b)?