ST430 Spring 2007 **Midterm 2** April 6, 2007, 11:00am–11:50am

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There are four questions, with a total of 45 points plus 5 Extra Credit points (Problem 3(d)). **Please start a new page for each question and do not use both sides.**

1. [Total 10 points]

   If $H_0 : \mu = 150$ is to be tested against $H_1 : \mu < 150$ at the $\alpha = 0.10$ level of significance based on a random sample of size n from a normal distribution with $\sigma = 15.0$, what is the smallest value for n that will make the power equal to at least 0.75 when $\mu = 147$?

2. [Total 10 points]

   Let Y, in millimeters, be the growth in 15 days of a tumor induced in a mouse. Assume that the distribution of Y is $N(\mu, \sigma^2)$. Nine measurements were obtained, which give $\bar{y} = 4.3$ and $s = 1.2$.

   (a) [5] Perform a suitable hypothesis test for $H_0 : \mu = 4.0$ against $H_1 : \mu \neq 4.0$ at a significance level of $\alpha = 0.10$.

   (b) [5] It is claimed that the standard deviation is greater than 1.0. Test this claim ($H_0 : \sigma = 1.0$ versus $H_1 : \sigma > 1.0$) at significance level $\alpha = .10$.

3. [Total 15 points + 5 extra credit points]

   Let $Y_1, \ldots, Y_{10}$ be a random sample from the pdf $f_Y(y;\theta) = \frac{1}{\theta} e^{-\frac{y}{\theta}}$, $y > 0$, $\theta > 0$.

   (a) [5] Show that $\frac{2}{\theta} Y_1$ has a chi-square distribution with 2 degrees of freedom. What is the distribution of $\frac{2}{\theta}(Y_1 + \cdots + Y_{10})$?

   (b) [5] Consider the critical region for testing $H_0 : \theta = 2$ versus $H_1 : \theta > 2$ defined by $Y_1 + Y_2 + \cdots + Y_{10} > c$. Determine c in order to achieve a .05 level test.

   (c) [5] For the test described in part (b), find the power of the test when $\theta = 2.21$. (Hint: Use result from part (a). If necessary, you can use rough approximations from the tables supplied.)

   (d) [**5 points EXTRA CREDIT**] Find the form of the GLRT for testing $H_0 : \theta = 2$ versus $H_1 : \theta > 2$ and show that it is equivalent to the test given in (b). That is, the test rejects H_0 in favor of H_1 if $\sum_{i=1}^{10} Y_i > c$.

4. [Total 10 points]

   Let $Y_1, \ldots, Y_5$ be IID with distribution $N(0, 4)$.

   (a) [5] Calculate $P\{(Y_1^2 + Y_2^2) < 3.64(Y_3^2 + Y_4^2 + Y_5^2)\}$.

   (b) [5] Calculate $P\{Y_1 + Y_2 < 4.353\,|Y_3|\}$. (Hint: $|Y_3| = \sqrt{Y_3^2}$.)