ST 430
Homework #12

11.3.1 $\beta_1 = \dfrac{4(93)-10(40.2)}{4(30)-10^2} = -1.5$

$\beta_0 = \dfrac{(40.2)-(-1.5)(10)}{4} = 13.8$

Thus, $y = 13.8 - 1.5x$. $t = \dfrac{\hat{\beta}_1 - \beta_1^0}{s\Big/\sqrt{\sum_{i=1}^{4}(x_i-\bar{x})^2}} = \dfrac{-1.5-0}{2.114/\sqrt{5}} = -1.59$

Since $-t_{.025,2} = -4.3027 < t = -1.59 < 4.3027 = t_{.025,2}$, accept H_0.

11.3.2 (a) The radius of the confidence interval =

$t_{.025,11}\dfrac{s}{\sqrt{\sum_{i=1}^{13}(x_i-\bar{x})^2}} = 2.2010\dfrac{42.745}{\sqrt{4602525.692}} = 0.044$

The center is $\beta_1 = 0.055$, and the confidence interval is (0.011, 0.099)

(b) Since 0 is not in the confidence interval, we can reject H_0 at the 0.05 level of significance.

(c) See the solution to Question 11.2.7. The linear fit for x values less than \$4300 is not very good, suggesting a search for other contributing variables in the x range of \$3500 to \$4200.

11.3.3 $t = \dfrac{\beta_1 - \beta_1^0}{s\Big/\sqrt{\sum_{i=1}^{15}(x_i-\bar{x})^2}} = \dfrac{3.291-0}{3.829/\sqrt{40.55733}} = 5.47.$

Since $t = 5.47 > t_{0.005,13} = 3.0123$, reject H_0.

11.3.9 $t = \dfrac{\hat{\beta}_1 - \beta_1^0}{s\Big/\sqrt{\sum_{i=1}^{11}(x_i-\bar{x})^2}} = \dfrac{0.84-0}{2.404/\sqrt{156.909}} = 4.38$

Since $t = 4.38 > t_{.025,9} = 2.2622$, reject H_0.

11.3.10 $E(\overline{Y}) = \dfrac{1}{n}\sum_{i=1}^{n}E(Y_i|x_i) = \dfrac{1}{n}\sum_{i=1}^{n}(\beta_0+\beta_1 x_i) = \dfrac{1}{n}n\beta_0 + \beta_1\dfrac{1}{n}\sum_{i=1}^{n}x_i = \beta_0 + \beta_1\bar{x}$

11.3.16 (a) The radius of the confidence interval is $t_{.25,16}s\sqrt{\frac{1}{n}+\frac{(x-\overline{x})^2}{\sum_{i=1}^{18}(x_i-\overline{x})^2}}$

$= 2.1199(0.202)\sqrt{\frac{1}{18}+\frac{(14.0-15.0)^2}{96.38944}} = 0.110$

The center is $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = -0.104 + 0.988(14.0) = 13.728$.

The confidence interval is (13.62, 13.84)

(b) The radius of the prediction interval is

$t_{.025,16}s\sqrt{1+\frac{1}{n}+\frac{(x-\overline{x})^2}{\sum_{i=1}^{18}(x_i-\overline{x})^2}} = 2.1199(0.202)\sqrt{1+\frac{1}{18}+\frac{(14.0-15.0)^2}{96.38944}} = 0.442$

The center is $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = -0.104 + 0.988(14.0) = 13.728$.

The confidence interval is (13.29, 14.17)