ST 430
Spring 2007
Homework #5

Solutions

5.3.18 From Definition 5.3.1, $d = \dfrac{1.96}{2\sqrt{202}} = 0.069$. The sample proportion is $86/202 = 0.426$. The largest believable value is $0.426 + 0.069 = 0.495$, so we should not accept the notion that the true proportion is as high as 50%.

5.3.24 Take n to be the smallest integer $\geq \dfrac{z_{.005}^2 p(1-p)}{(0.05)^2} = \dfrac{2.58^2(0.40)(0.60)}{(0.05)^2} = 639.01$, so $n = 640$.

5.3.26 a) Take n to be the smallest integer $\geq \dfrac{z_{.075}^2}{4(0.03)^2} = \dfrac{1.44^2}{4(0.03)^2} = 576$.

b) Take n to be the smallest integer $\geq \dfrac{z_{.075}^2 p(1-p)}{(0.03)^2} = \dfrac{1.44^2(0.10)(0.90)}{(0.03)^2} = 207.36$, so let $n =$ 208.

6.2.2 Let μ = true average IQ of students after drinking Brain-Blaster. To test H_0: $\mu = 95$ versus H_1: $\mu \neq 95$ at the $\alpha = 0.06$ level of significance, the null hypothesis should be rejected if $z = \dfrac{\bar{y} - 95}{15/\sqrt{22}}$ is either 1) ≤ -1.88 or 2) ≥ 1.88. Equivalently, H_0 will be rejected if $\bar{y}$ is either **1)** $\leq 95 - (1.88)\dfrac{15}{\sqrt{22}} = 89.0$ or **2)** $\geq 95 + (1.88)\dfrac{15}{\sqrt{22}} = 101.0$.

6.2.4 Assuming there is no reason to suspect that the polymer would shorten a tire's lifetime, the alternative hypothesis should be H_1: $\mu > 32{,}500$. At the $\alpha = 0.05$ level, H_0 should be rejected if the test statistic exceeds $z_{.05} = 1.64$. But $z = \dfrac{33{,}800 - 32{,}500}{4000/\sqrt{15}} = 1.26$, implying that the observed mileage increase is not statistically significant.

6.2.6 By definition, $\alpha = P(29.9 \le \bar{Y} \le 30.1 \mid H_0 \text{ is true}) = P\left(\frac{29.9-30}{6.0/\sqrt{16}} \le \frac{\bar{Y}-30}{6.0/\sqrt{16}} \le \frac{30.1-30}{6.0/\sqrt{16}}\right) =$ $P(-0.07 \le Z \le 0.07) = 0.056$. The interval (29.9, 30.1) is a poor choice for C because it rejects H_0 for the $\bar{y}$ - values that are most compatible with H_0 (that is, closest to $\mu_0 = 30$). Since the alternative is two-sided, H_0 should be rejected if $\bar{y}$ is either

1) $\le 30 - 1.91 \cdot \frac{6.0}{\sqrt{16}} = 27.1$ or **2)** $\ge 30 + 1.91 \cdot \frac{6.0}{\sqrt{16}} = 32.9$.

6.2.10 Let μ = true average blood pressure when taking statistics exams. Test H_0: $\mu = 120$ versus H_1: $\mu > 120$. Given that $\sigma = 12$, $n = 50$ and $\bar{y} = 125.2$, $z = \frac{125.2-120}{12/\sqrt{50}} = 3.06$. The corresponding P-value is approximately 0.001 ($= P(Z \ge 3.06)$), so H_0 would be rejected for any usual choice of α.

6.3.4 The null hypothesis would be rejected if $z = \frac{k - 200(0.45)}{\sqrt{200(0.45)(0.55)}} \ge 1.08$ ($= z_{.14}$). For that to happen, $k \ge 200(0.45) + 1.08 \cdot \sqrt{200(0.45)(0.55)} \doteq 98$.

6.3.6 Let $p = P$(person dies if month preceding birthmonth). Test H_0: $p = \frac{1}{12}$ versus H_1: $p < \frac{1}{12}$. Given that $\alpha = 0.05$, H_0 should be rejected if $z \le -1.64$. In this case, $z = \frac{16 - 348(1/12)}{\sqrt{348(1/12)(11/12)}} =$ -2.52, which suggests that people do not necessarily die randomly with respect to the month in which they were born. More specifically, there appears to be a tendency to "postpone" dying until the next birthday has passed.

6.4.4 For $n = 16$, $\sigma = 4$, and $\alpha = 0.05$, H_0: $\mu = 60$ should be rejected in favor of a two-sided H_1 if either **1)** $\bar{y} \le 60 - 1.96 \cdot \frac{4}{\sqrt{16}} = 58.04$ or **2)** $\bar{y} \ge 60 + 1.96 \cdot \frac{4}{\sqrt{16}} = 61.96$. Then, for arbitrary μ, $1 - \beta = P(\bar{Y} \le 58.04 \mid \mu) + P(\bar{Y} \ge 61.96 \mid \mu)$. Selected values of $(\mu, 1 - \beta)$ that would lie on the power curve are listed in the accompanying table.

μ	$1 - \beta$
56	0.9793
57	0.8508
58	0.5160
59	0.1700
60	0.05 ($=\alpha$)
61	0.1700
62	0.5160
63	0.8508
64	0.9793