ST 430
Homework #8 Solutions

7.4.2 a) 2.508 b) −1.079 c) 1.7056 d) 4.3027

7.4.4 Since $\frac{\bar{Y}-27.6}{S/\sqrt{9}}$ is a Student t random variable with 8 df, $P\left(-1.397 \le \frac{\bar{Y}-27.6}{S/\sqrt{9}} \le 1.397\right) =$ 0.80 and $P\left(-1.8595 \le \frac{\bar{Y}-27.6}{S/\sqrt{9}} \le 1.8595\right) = 0.90$ (see Appendix Table A.2).

7.4.6 $P(90.6 - k(S) \le \bar{Y} \le 90.6 + k(S)) = 0.99 =$
$P\left(\frac{90.6-k(S)-90.6}{S/\sqrt{20}} \le \frac{\bar{Y}-90.6}{S/\sqrt{20}} \le \frac{90.6+k(S)-90.6}{S/\sqrt{20}}\right) = P\left(\frac{k(S)}{S/\sqrt{20}} \le T_{19} \le \frac{k(S)}{S/\sqrt{20}}\right) =$
$P(-2.8609 \le T_{19} \le 2.8609)$, so $\frac{k(S)}{S/\sqrt{20}} = 2.8609$, implying that $k(S) = \frac{2.8609 \cdot S}{\sqrt{20}}$.

7.4.8 Given that $n = 7$, $t_{\alpha/2,n-1} = t_{.025,6} = 2.4469$. Here $\sum_{i=1}^{n} y_i = 12{,}808$ and $\sum_{i=1}^{n} y_i^2 = 26{,}540{,}436$ so $\bar{y}$ $= \frac{1}{7}(12{,}808) = 1829.71$ and $s = \sqrt{\frac{7(26{,}540{,}436)-(12{,}808)^2}{7(6)}} = 719.43$.
The confidence interval is $\left(1829.71 - 2.4469\frac{719.43}{\sqrt{7}}, 1829.71 + 2.4469\frac{719.43}{\sqrt{7}}\right)$
= (\$1164.35, \$2495.07).

7.4.10 Let μ = true average daily fat intake of males in the age group 25 to 34. Since $\bar{y} = \frac{1}{10}(1101.3) = 110.13$, $s = \sqrt{\frac{10(128{,}428.67)-(1101.3)^2}{10(9)}} = 28.17$, and $t_{.05,9} = 1.8331$, the 90% confidence interval for μ is $\left(110.13 - 1.8331 \cdot \frac{28.17}{\sqrt{10}}, 110.13 + 1.8331 \cdot \frac{28.17}{\sqrt{10}}\right)$, which reduces to (93.80, 126.46).

7.4.12 Given that $n = 16$, $t_{\alpha/2,n-1} = t_{.025,15} = 2.1315$, so $\left(\bar{y} - 2.1315 \cdot \frac{s}{\sqrt{16}}, \bar{y} + 2.1315 \cdot \frac{s}{\sqrt{16}}\right) =$ (44.7, 49.9). Therefore, $49.9 - 44.7 = 5.2 = 2(2.1315) \cdot \frac{s}{\sqrt{16}}$, implying that $s = 4.88$. Also, because the confidence interval is centered around the sample mean, $\bar{y} = \frac{44.7+49.9}{2} = 47.3$.

7.4.19 Let μ = true average GMAT increase earned by students taking the review course. The hypotheses to be tested are H_0: $\mu = 40$ versus H_1: $\mu < 40$. Here, $\sum_{i=1}^{15} y_i = 556$ and $\sum_{i=1}^{15} y_i^2 = 20{,}966$, so $\bar{y} = \frac{556}{15} = 37.1$, $s = \sqrt{\frac{15(20{,}966)-(556)^2}{15(14)}} = 5.0$, and $t = \frac{37.1-40}{5.0/\sqrt{15}} = -2.25$. Since $-t_{.05,14} = -1.7613$, H_0 should be rejected at the $\alpha = 0.05$ level of significance, suggesting that the MBAs 'R Us advertisement may be fraudulent.

7.5.2 a) 0.95 b) 0.90 c) $0.975 - 0.025 = 0.95$
d) 0.99

7.5.6 $P\left(\frac{S^2}{\sigma^2} < 2\right) = P\left(\frac{(n-1)S^2}{\sigma^2} < 2(n-1)\right) = P\left(\chi^2_{n-1} < 2(n-1)\right)$. Values from the 0.95 column in a χ^2 table show that for each $n < 8$, $P\left(\chi^2_{n-1} < 2(n-1)\right) < 0.95$. But for $n = 9$, $\chi^2_{.95,8} = 15.507$, which means that $P\left(\chi^2_8 < 16\right) > 0.95$.

7.5.8 If $n = 19$ and $\sigma^2 = 12.0$, $\frac{18S^2}{12.0}$ has a χ^2 distribution with 18 df, so

$$P\left(8.231 \le \frac{18S^2}{12.0} \le 31.526\right) = 0.95 = P(5.49 \le S^2 \le 21.02).$$