ST 430
Homework #9: Solutions

9.2.2 $s_p = \sqrt{\dfrac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}} = \sqrt{\dfrac{3(267^2)+3(224^2)}{4+4-2}} = 246.44$

$t = \dfrac{\bar{x}-\bar{y}}{s_p\sqrt{1/n+1/m}} = \dfrac{1133.0-1013.5}{246.44\sqrt{1/4+1/4}} = 0.69$

Since $-t_{.025,6} = -2.4469 < t = 0.69 < t_{.025,6} = 2.4469$, accept H_0.

9.2.4 $s_p = \sqrt{\dfrac{5(15.1^2)+8(8.1^2)}{6+9-2}} = 11.317$

$t = \dfrac{70.83-79.33}{11.317\sqrt{1/6+1/9}} = -1.43$

Since $-t_{.005,13} = -3.0123 < t = -1.43 < t_{.005,13} = 3.0123$, accept H_0.

9.2.8 The solution given in the manual is incorrect. The means and standard deviations are given in different units, which must be adjusted so that the t statistic is unitless. This solution converts all the units to minutes. Alternately, all units could be converted to hours.

H_0: $u_x – 1 = u_y$
H_1: $u_x – 1 < u_y$

s_p = sqrt((10 * 12^2 + 10 * 16^2)/(10 + 10 – 2)) = sqrt(200) = 14.1421

t = ((2.1 – 1 – 1.6) * 60)/(14.1421 * sqrt(1/10 + 1/10)) = -4.743

Reject H_0 if t < -$t_{0.05,18}$ = -1.7341   t < -$t_{0.05,18}$ Reject H_0 and conclude H_1.

9.2.9 a) Reject H_0 if $t > t_{.005,15} = 2.9467$, so we seek the smallest value of $|\bar{x}-\bar{y}|$ such that

$t = \dfrac{|\bar{x}-\bar{y}|}{s_p\sqrt{1/n+1/m}} = \dfrac{|\bar{x}-\bar{y}|}{15.3\sqrt{1/6+1/11}} > 2.9467$, or $|\bar{x}-\bar{y}| > (15.3)(0.508)(2.9467)$
$= 22.90$

b) Reject H_0 if $t > t_{.05,19} = 1.7291$, so we seek the smallest value of $\bar{x}-\bar{y}$ such that

$t = \dfrac{\bar{x}-\bar{y}}{s_p\sqrt{1/n+1/m}} = \dfrac{\bar{x}-\bar{y}}{214.9\sqrt{1/13+1/8}} > 1.7291$, or $\bar{x}-\bar{y} > (214.9)(0.44936)(1.7291)$
$=166.97$

9.3.4 The observed $F = 3.18^2/5.67^2 = 0.315$. Since $F_{.025,9,9} = 0.248 < 0.315 < 4.03 = F_{.975,9,9}$, we can accept H_0 that the variances are equal.

9.3.6 The observed $F = 398.75/274.52 = 1.453$. Let $\alpha = 0.05$. The critical values are $F_{.025,13,11}$ and $F_{.975,13,11}$. These values are not in Table A.4, so approximate them by $F_{.025,12,11} = 0.301$ and $F_{.975,12,11} = 3.47$. Since $0.301 < 1.453 < 3.47$, accept H_0 that the variances are equal. Theorem 9.2.2 is appropriate.

9.4.2 $\hat{p} = \dfrac{x+y}{n+m} = \dfrac{66+93}{423+423} = 0.188$

$$z = \frac{\dfrac{x}{n} - \dfrac{y}{m}}{\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n} + \dfrac{\hat{p}(1-\hat{p})}{m}}} = \frac{\dfrac{66}{423} - \dfrac{93}{423}}{\sqrt{\dfrac{0.188(0.812)}{423} + \dfrac{0.188(0.812)}{423}}} = -2.38$$

For this experiment, H_0: $p_X = p_Y$ and H_1: $p_X < p_Y$. Since $z = -2.38 < -1.64 = -z_{.05}$, reject H_0.

9.4.4 $\hat{p} = \dfrac{53+705}{91+1117} = 0.627$

$$z = \frac{\dfrac{53}{91} - \dfrac{705}{1117}}{\sqrt{\dfrac{0.627(0.373)}{91} + \dfrac{0.627(0.373)}{1117}}} = -0.92$$

Since $-2.58 < z = -0.92 < 2.58 = z_{.005}$, accept H_0 at the 0.01 level of significance.

9.4.6 $\hat{p} = \dfrac{2915+3086}{4134+4471} = 0.697$

$$z = \frac{\dfrac{2915}{4134} - \dfrac{3086}{4471}}{\sqrt{\dfrac{0.697(0.303)}{4134} + \dfrac{0.697(0.303)}{4471}}} = 1.50$$

Since $-1.96 < z = 1.50 < 1.96 = z_{.025}$, accept H_0 at the 0.05 level of significance.

9.4.8 $\hat{p} = \dfrac{78+50}{300+200} = 0.256$

$$z = \frac{\dfrac{78}{300} - \dfrac{50}{200}}{\sqrt{\dfrac{0.256(0.744)}{300} + \dfrac{0.256(0.744)}{200}}} = 0.25.$$ In this situation, H_1 is $p_X > p_Y$.

Since $z = 0.25 < 1.64 = z_{.05}$, accept H_0. The player is right.

9.5.2 The center of the confidence interval is $\bar{x} - \bar{y} = 6.7 - 5.6 = 1.1$. $s_p = \sqrt{\dfrac{8(0.54^2) + 6(0.36^2)}{14}} =$ 0.47. The radius is $t_{\alpha/2,n+m-2}s_p\sqrt{\dfrac{1}{n}+\dfrac{1}{m}} = 1.7613(0.47)\sqrt{\dfrac{1}{9}+\dfrac{1}{7}} = 0.42$. The confidence interval is (1.1 – 0.42, 1.1 + 0.42) = (0.68, 1.52). Since 0 is not in the interval, we can reject the null hypothesis that $\mu_X = \mu_Y$.

9.5.8 The confidence interval is $\left(\dfrac{s_X^2}{s_Y^2}F_{.025,5,7}, \dfrac{s_X^2}{s_Y^2}F_{.975,5,7}\right) = \left(\dfrac{137.4}{340.3}(0.146),\ \dfrac{137.4}{340.3}(5.29)\right)$

= (0.06, 2.14)

Since the confidence interval contains 1, we can accept H_0 that the variances are equal, and Theorem 9.2.1 applies.

9.5.12 The center of the confidence interval is $\dfrac{x}{n} - \dfrac{y}{m} = \dfrac{106}{3522} - \dfrac{13}{115} = -0.083$. The radius is

$$z_{.025}\sqrt{\frac{\left(\frac{x}{n}\right)\left(1-\frac{x}{n}\right)}{n} + \frac{\left(\frac{y}{m}\right)\left(1-\frac{y}{m}\right)}{m}} = 1.97\sqrt{\frac{\left(\frac{106}{3522}\right)\left(1-\frac{106}{3522}\right)}{3522} + \frac{\left(\frac{13}{115}\right)\left(1-\frac{13}{115}\right)}{115}} = 0.058$$

The 95% confidence interval is (–0.083 – 0.058, –0.083 + 0.058) = (–0.141, –0.025)

Since the confidence interval lies to the left of 0, there is statistical evidence that the suicide rate among women members of the American Chemical Society is higher.