Chapter Seven - Part I
The Normal Probability Distribution

GOALS
When you have completed this lecture, you will be able to:

ONE
List the characteristics of the normal probability distribution.
TWO
Begin to use table of areas under the normal curve
THREE
Define and calculate z values, and use them to determine the relative location of a value.

Continuous Random Variables
- A continuous random variable can assume any value in an interval on the real line or in a collection of intervals.
- Recall that for a discrete random variable, we computed P(X).
  - Ex: X=number of days in a year with wind speeds above 50 mph,
    P(19)=?
    P(19 or 20)=?

Continuous Random Variables
- For a continuous random variable, it is not possible to talk about the probability of the random variable assuming a particular value.
- Instead, we can compute the probability that a random variable will assume a value within a given interval
  - Ex: X=wind speed on a randomly selected day.
  - What is the probability that X is larger than 50?
  - What is the probability that X is between 10 and 50?

Continuous Probability Distributions
- Continuous probability distributions are models for continuous random variables.
- To represent all possible outcomes and their associated probabilities:
  - For discrete distributions, we draw a histogram
  - For continuous distributions, we draw a smooth curve
- The total area under the curve must equal 1, or 100%.
Normal Distribution

- For many continuous random variables, we can assume the random variable follows a normal distribution.
- A normal distribution is bell-shaped, symmetric, has mean $\mu$ and standard deviation $\sigma$.

**EXAMPLE 1**

The monthly incomes of recent MBA graduates in a large corporation are known to have a bell-shaped distribution with a mean of $3000$ and a standard deviation of $200$.

A bell-shaped distribution of values is also referred to as a *normal* distribution.
Areas Under the Normal Curve

- Recall the Empirical Rule,
  - about 68% of the area under the normal curve is within one standard deviation of the mean. $\mu \pm 1\sigma$
  - about 95% is within two standard deviations of the mean. $\mu \pm 2\sigma$
  - About 99.7% is within three standard deviations of the mean. $\mu \pm 3\sigma$

EXAMPLE 1

For the recent MBA graduates, $\mu = 3000$ and $\sigma = 200$. About 95% of the MBA graduates earn between what two values?

$$\mu \pm 2\sigma =$$

What is the probability that a recent MBA graduate will earn
- Less than $3000$?
- More than $3200$?
- Between $3000$ and $3300$?
The Standard Normal Probability Distribution

- A normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$ is called the standard normal distribution.
- A continuous random variable that follows a standard normal distribution is usually denoted by $Z$.
- Areas for the standard normal distribution (probabilities) are listed in Appendix D of your textbook and are also given on the inside back cover of the textbook.

Example 2: areas under the standard normal curve

- If $Z$ follows a standard normal distribution:
  - What is the probability that $Z$ is between 0 and 1.96?
  - What is the probability that $Z$ is between 0 and 1.54?

Example 2, continued

- What is the probability that $Z$ is larger than 1.96?
- What is the probability that $Z$ is between $-1.96$ and 0?
- What is the probability that $Z$ is between $-1$ and 1?

Using the Standard Normal Probability Distribution to find areas when $\mu \neq 0$ and/or $\sigma \neq 1$

- If $X$ follows a normal distribution with mean $\mu$ and standard deviation $\sigma$, we can determine the number of standard deviations $X$ is above or below $\mu$:

$$ Z = \frac{X - \mu}{\sigma} $$

- Thus we can covert
  - $X$ which has mean $\mu$ and standard deviation $\sigma$
  - $Z$ which has mean 0 and standard deviation 1 (standard normal).
Z Values

- The Z value indicates the number of standard deviations a value is above or below the mean of the distribution.
  - An observation with a z value of 2.5 would be 2.5 standard deviations greater than the mean.
  - An observation with a z value of -1.3 would be 1.3 standard deviations less than the mean.

EXAMPLE 1

For the monthly incomes of the recent MBA graduates, what is the Z value for an income of $3200? An income of $2700?

- For X = $3200,
- For X = $2700,

EXAMPLE 1 continued

- A Z value of 1 indicates that the value of $3200 is __________________________

- A Z value of -1.5 indicates that the value of $2700 is __________________________

First part of Textbook Assignment 4: Page 228 - #1,4, Page 230 - #6(a).