Chapter Eight - Part V
Sampling Methods and Sampling Distributions

GOALS
When you have completed this lecture, you should be able to:

ONE
Calculate confidence intervals for population proportions.
TWO
Explain when and how to apply a finite population correction factor.
THREE
Determine the sample size needed to estimate a mean to within a given margin of error at a given level of confidence.
FOUR
Determine the sample size needed to estimate a proportion to within a given margin of error at a given level of confidence.

EXAMPLE 1
- A study was done to investigate behavior of college students. 500 college students were randomly selected for a survey. 340 of the students stated that they have not driven a car under the influence of alcohol or other drugs.
- Compute a confidence interval estimate for the proportion of college students who have not driven a car while “under the influence.”
- Let $\pi$ represent the population proportion of college students who have not driven a car while under the influence.
- Then $p =$ is a point estimate for $\pi$.

Confidence Interval for a Population Proportion

- The confidence interval for a population proportion, $\pi$, is given by:
  \[ p \pm z \sigma_p \]
- where $\sigma_p$ is the standard error of the proportion:
  \[ \sigma_p = \sqrt{\frac{p(1-p)}{n}} \]
- and $p$ is the proportion obtained for a simple random sample from the population.

EXAMPLE 1 continued
- Here, $n = \_\_\_\_\_, p = \_\_\_\_\_\_, and z = \_\_\_\_\_
- The standard error of the proportion is
  \[ \sigma_p = \sqrt{\frac{p(1-p)}{n}} = \]
- the 98% CI is
  \[ p \pm z \sigma_p = \]
- Interpretation:
**Finite-Population Correction Factor**

- The formulas given for confidence intervals so far have assumed an infinite or very large population.
- For a finite population, where the total number of objects is N and the size of the sample is n, the following adjustment can be made to the standard errors of the sample means and the proportion:
  - Standard error of the sample means:
    \[ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} \]
  - Standard error of the sample proportions:
    \[ \sigma_p = \sqrt{\frac{p(1-p)}{n}} \sqrt{\frac{N-n}{N-1}} \]
- This adjustment is called the *finite-population correction factor.*
- Note: If \( n/N < .05 \), the finite-population correction factor is usually ignored.

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**EXAMPLE 2**

- Suppose that a sample of 100 items is obtained from a shipment of 1000 items, in order to estimate the proportion of defective items, \( \pi \).
- The sample results showed 8 items to be defective.
- The point estimate for \( \pi \) is \( \hat{\pi} = \frac{8}{100} \).
- Construct a 99% confidence interval for the true proportion of defective items.
- Since \( n/N = 100/1000 = .10 > .05 \), we have to use the finite population correction factor.

\[ \sigma_p = \sqrt{\frac{p(1-p)}{n}} \sqrt{\frac{N-n}{N-1}} \]

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**EXAMPLE 2 continued**

- For 99% confidence, we have \( z = \)
- The resulting confidence interval is
- Interpretation:
Selecting a Sample Size

- There are 3 factors that determine the size of a sample:
  - The degree of confidence selected.
  - The maximum allowable error.
  - The variation of the population.

Variation in the Population

- Sample Size for Estimating a Mean: A convenient computational formula for determining n is:
  \[ n = \left( \frac{Z \cdot S}{E} \right)^2 \]
- where:
  - E is the allowable error,
  - Z is the z score associated with the degree of confidence selected, and
  - S is the sample deviation from a pilot study.

Example 2

- A consumer group would like to estimate the mean monthly electric bill for 2 bedroom apartments in February. Based on similar studies the standard deviation is estimated to be $20.00. A 99% level of confidence is desired, with an accuracy of $\pm 5.00$. How large a sample is required?

\[ n = \left( \frac{Z \cdot S}{E} \right)^2 = \]

Sample Size for Proportions

- The formula for determining the sample size in the case of a proportion is:
  \[ n = p (1 - p) \left( \frac{Z}{E} \right)^2 \]
- where:
  - p is the estimated proportion, based on past experience or a pilot survey (.5 is used if no information is available);
  - Z is the z value associated with the degree of confidence selected;
  - E is the maximum allowable error the researcher will tolerate.
EXAMPLE 4

- The American Kennel Club wanted to estimate the proportion of children that have a dog as a pet. If the club wanted the estimate to be within 3% of the population proportion, how many children would they need to contact? Assume a 95% level of confidence.

\[ n = p (1 - p) \left( \frac{Z}{E} \right)^2 = \]

- Now assume that a pilot study had indicated that approximately 30% of children have a dog as a pet. Compute a 95% confidence interval.

End of Chapter 8

- Exam 2 on Friday, so no Textbook Assignment this week.
- Recommended practice problems:
  - Page 287: #23
  - Page 293: #31, 35.
- Don’t forget: Minitab lab #2 due Thursday!