Conditional Probability

the prob. of event A occurring given that event B has already occurred
denoted $P(A|B)$

*Example*

ask 10 people their age and gender

$A =$ person is female
$B =$ in twenties
$C =$ in thirties

What proportion of the circle contains A?

What is the probability the person is female, if we know in their twenties?

$$P(A|B) = \frac{2}{6} = \frac{1}{3}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{2}{10}$$

$$= \frac{2 \cdot 10}{10} = \frac{2}{6}$$

$$= \frac{1}{3}$$
Independence

**def:** events \( A \) and \( B \) are said to be independent if the occurrence of event \( A \) has no effect on the probability of event \( B \) occurring (and vice versa).

So... if \( A \) and \( B \) are independent (written \( A \perp B \)) then,

\[
P(A|B) = P(A) \quad \text{and} \quad P(B|A) = P(B)
\]

**example 1:** Fair coin toss

2 events: \( T \) or \( H \)

Toss coin once

\[
P(T|H) = \_? \quad P(T) = \frac{1}{2}
\]

\[
= 0 \quad \Rightarrow \quad P(T|H) \neq P(T) \quad \text{not independent!}
\]

**example 2:**

\[
\begin{array}{c}
\text{A} = \text{girl} \\
\text{B} = \text{20's} \\
\text{C} = \text{30's}
\end{array}
\]

Are \( A \) and \( C \) independent?

\[
P(A|C) = 0
\]

\[
P(A) = \frac{3}{10}
\]

\[
P(A|C) \neq P(A) \Rightarrow \text{not independent!}
\]

\[
P(B|C) = P(B)
\]
5) The multiplication rule

\[ P(A \cap B) = P(A|B) \cdot P(B) \]

by the earlier fact \( P(A|B) = \frac{P(A \cap B)}{P(B)} \)

so... this tells us that

if A and B are independent

then \( P(A \cap B) = P(A) \cdot P(B) \)

yesterday’s example • toss a fair coin twice

\[ P(\text{two heads}) = P(\text{H on 1st toss}) \cdot P(\text{H on 2nd toss}) \]

\[ = \frac{1}{2} \cdot \frac{1}{2} \]

\[ = \frac{1}{4} \]

• def: \)

events A and B are disjoint if \( P(A \cap B) = 0 \),

ie A and B can not occur at the same time
Example: Medical Testing (pg 87)

Positive and Negative test results

8% of some population has a disease

If a person has a disease, 95% chance of positive
If a person does not have, 90% chance of negative

\[
\begin{align*}
\text{True } + & = (0.08)(0.95) = 0.076 \\
\text{False } - & = (0.08)(0.05) = 0.004 \\
\text{False } + & = (0.92)(0.1) = 0.092 \\
\text{True } - & = (0.92)(0.9) = 0.828
\end{align*}
\]

\[
P(\text{test positive}) = 0.076 + 0.092 = 0.168
\]

If someone tests positive, what is the chance that the person really has the disease?

\[
P(\text{have disease} | \text{test positive}) = \frac{P(\text{have disease} \land \text{test positive})}{P(\text{test positive})} = \frac{0.076}{0.168} \approx 0.452
\]
Random Variables

- **def**: a **random variable** is a variable that takes on a value that depends on the outcome of a chance observation.

Ex. toss a coin

say \( X = \) the face that shows up

\[ = \{ H, T \} \]