Applications of the Binomial Distribution: Antibiotic Resistance

Use the information in the following setting to answer questions 1 through 11:
One of the downsides of treating bacterial infections with an antibiotic is that, unless the
patient follows the prescription protocol for the entire time of treatment, there is a very high
probability that a certain proportion of the bacteria population will survive. These resistant
microbes can then pass on this resistance to subsequent generations. In 2004 8.3% of
the gonorrhea cases seen in Denver tested positive for full resistance to ciprofloxacin. By
2010 the proportion of ciprofloxacin resistant cases had risen to 14.2%.
[http://www.cdc.gov/std/Gonorrhea/arg/basic.htm
http://www.cdc.gov/std/gisp2010/figI.htm#a2]

Consider a clinician who sees 10 cases of gonorrhea in the course of a week and let 'X' be
the number of these cases that test positive for ciprofloxacin resistance (year = 2004)

1) The type/level for the variable of interest in this setting is:
   variable = whether or not this strain is resistant to ciprofloxacin.
categorical variable yes/no?, nominal

2) Write out the sample space for the variable 'X'
   \[ X = \{0, 1, 2, \ldots, 10\} \]

3) The probability that exactly two of patients from this group test positive for ciprofloxacin
   resistance is:
   \[ P(X = 2) = \binom{10}{2} (0.083)^2 (0.917)^8 = \frac{10!}{2!8!} (0.083)^2 (0.917)^8 = 0.155 \]

4) \[ P(X = 8) = \binom{10}{8} (0.083)^8 (0.917)^2 = \frac{10!}{8!2!} (0.083)^8 (0.917)^2 = 0.000000852 \approx 8.52 \times 10^{-8} \approx 0 \]

5) The probability that none of the patients test positive for ciprofloxacin resistance is:

6) The probability that all 10 of the patients test positive for ciprofloxacin resistance is:

7) \[ P(X \leq 5) = P(X=0) + P(X=1) + P(X=2) + \ldots + P(X=5) = 0.42 + .381 + .155 + .0374 + .0059 + .00064 = 0.99994 \]
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8) $P(X < 5) = P(X \leq 5) - P(X = 5)$
   $= .99994 - .00064$
   $= .9993$

9) The probability that, at least 3 of the 10 patients test positive for ciprofloxacin resistance is:
   $P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$
   $= 1 - (.420 + .381 + .155) = 1 - .956$
   $= .044$

10) Write out the probability mass function (PMF) for the number of patients who test positive for ciprofloxacin resistance:

<table>
<thead>
<tr>
<th>$X$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X=x)$</td>
<td>.42</td>
<td>.381</td>
<td>.155</td>
<td>.034</td>
<td>~0</td>
<td>~0</td>
<td>~0</td>
<td>~0</td>
<td>~0</td>
<td>~0</td>
<td>~0</td>
</tr>
</tbody>
</table>

11) The characteristics (assumptions) that allow us to appropriately use the binomial distribution to answer questions 3 through 9 are:

1) Fixed number of observations. If it were random, we would use a different technique.

2) All observations in the sample are independent of one another.

3) Each observation falls into either success or failure

4) The probabilities remain constant from trial to trial