Random Variables

**Def:** a random variable is a variable that takes on a value that depends on the outcome of a chance observation.

ex. toss a coin
    say \( X = \) the face that shows up

\[ \text{Monday, January 28th} = \{ H, T \} \]

We say the r.v. \( X \) follows a distribution.
There are many known distributions.

For discrete r.v.'s, distributions are defined by the probability mass function (pmf).
- the pmf gives the probability of a r.v. at a given value

\[ f_Y(y) = P(Y = y) \]

**Example:** coin toss
  \( X = \) face that shows up
  \( = \{ H, T \} \)
  \[ f_X(H) = P(X = H) = \frac{1}{2} \]
  \[ f_X(T) = P(X = T) = \frac{1}{2} \]

\[ \sum_i f_Y(y_i) = 1 \]

\[ \sum_i f_X(x_i) = f_X(H) + f_X(T) = 1 \]
The Binomial Distribution

Suppose we have a sequence of \( n \) independent trials, each having a probability of success, \( p \), and a probability of failure \( q = 1 - p \).

We want to find the probability of getting \( j \) successes out of \( n \) trials (here \( j = 0, 1, \ldots, n \)).

\[
f_Y(j) = P(Y = j) = \frac{n!}{j!(n-j)!} \ p^j \ (1-p)^{n-j}
\]

\[
= \binom{n}{j} p^j q^{n-j}
\]

"!" is called **factorial**

\[
n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1
\]

So,

\[
6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1
\]

and a note: \( 0! = 1 \)