Goodness of Fit: Flower Offspring Color

Use the information in the following setting to answer questions 1 and 2.
We expect based on Mendelian genetics that if we breed two plants that have a Bb genotype (B is dominant purple flower, b is recessive white flower) that the offspring will have a ratio of 3 purple flowers for every 1 white flower. In a sample of 87 offspring, we get 58 purple flowers and 29 white flowers.

<table>
<thead>
<tr>
<th>Color</th>
<th>O_i</th>
<th>E_i</th>
<th>O_i - E_i</th>
<th>(O_i - E_i)^2</th>
<th>(O_i - E_i)^2/E_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purple</td>
<td>58</td>
<td>65.25</td>
<td>-7.25</td>
<td>52.5625</td>
<td>.8056</td>
</tr>
<tr>
<td>White</td>
<td>29</td>
<td>21.75</td>
<td>7.25</td>
<td>52.5625</td>
<td>2.4167</td>
</tr>
<tr>
<td>Total</td>
<td>87</td>
<td>87</td>
<td>0</td>
<td>X</td>
<td>3.2223</td>
</tr>
</tbody>
</table>

1) Finish filling in the above table.

2) Test the 3:1 ratio claim at the 0.10 level.

**Step 1**

\[ H_0: \pi_p = \frac{3}{4} = .75 \]
\[ H_a: \pi_p \neq \frac{3}{4} = .75 \]

**Step 2**

\[ \alpha = .10 \]

**Step 3**

assumptions hold since all E_i ≥ 5
Goodness of Fit: Flower Offspring Color

Continue with the rest of the hypothesis test even if the assumptions do not hold.

Step 4
\[ X^2 = 3.223 \]

Step 5
\[ \text{df} = k - 1 \]
\[ = 2 - 1 = 1 \]
\[ p - \text{value} = P( X^2 \geq 3.223 ) \in (0.05, 0.10) \]
\[ X^2_{\text{crit}} = 2.706 \]

Step 6
\[ p - \text{value} < \alpha \]
\[ \begin{align*}
& \text{Reject } H_0 \\
& X^2 > X^2_{\text{crit}}
\end{align*} \]

Step 7
We have enough evidence at the .10 significance level to reject that the flower offspring ratio is 3:1 purple to white.
Goodness of Fit: Dog Color Frequency

Use the information in the following setting to answer questions 1 and 2.
We believe that half of all dogs are brown, one-third of all dogs are black, and one-sixth
of all dogs are white. We take a random sample of 131 dogs and find that 71 are brown,
43 are black, and 17 are white.

<table>
<thead>
<tr>
<th>Color</th>
<th>O₁</th>
<th>E₁</th>
<th>O₁ - E₁</th>
<th>(O₁ - E₁)²</th>
<th>(O₁ - E₁)²/E₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brown</td>
<td>71</td>
<td>65.5</td>
<td>5.5</td>
<td>30.25</td>
<td>.4618</td>
</tr>
<tr>
<td>Black</td>
<td>43</td>
<td>43.67</td>
<td>-.67</td>
<td>.4489</td>
<td>.0103</td>
</tr>
<tr>
<td>White</td>
<td>17</td>
<td>21.83</td>
<td>-4.83</td>
<td>23.3289</td>
<td>1.0687</td>
</tr>
<tr>
<td>Total</td>
<td>131</td>
<td>131</td>
<td>0</td>
<td></td>
<td>Test Statistic</td>
</tr>
</tbody>
</table>

1) Finish filling in the above table.

2) Test the believed distribution at the 0.05 level.

Step 1

\[ \pi_{\text{brown}} = \frac{1}{2} \]

\[ \pi_{\text{black}} = \frac{1}{3} \]

\[ \pi_{\text{white}} = \frac{1}{6} \]

\[ H_0: \pi_{\text{brown}} = \frac{1}{2} \]

\[ H_a: \text{not } H_0 \]

Step 2

\[ \chi = 0.05 \]

Step 3

Assumptions hold since \( E_i \geq 5 \)
for \( i = 1, 2, 3 \)
**Goodness of Fit: Dog Color Frequency**

Continue with the rest of the hypothesis test even if the assumptions do not hold.

**Step 4**

\[ X^2 = 1.5408 \]

**Step 5**

\[ \text{p-value: } P(X^2 \geq 1.5408) > .1 \]

\[ (X^2_{\text{crit}} = 5.99) \]

**Step 6**

\[ \text{p-value} > .1 > .05 = \alpha \]

\[ X^2 = 1.54 < 5.99 = X^2_{\text{crit}} \]

\[ \text{FTR } H_0 \]

**Step 7**

We do not have enough evidence at the .05 significance level to conclude that dog color distribution is different than 1/2 brown, 1/3 black, 1/6 white.

In practice, we would treat the null distribution as the true distribution.