Wilcoxon Signed Rank Test: Nerve Cell Densities as Measured at Two Different Sites in the Small Intestine of Horses Example

Use the information in the following setting to answer questions 1 through 11

In a study designed to assess differences in nerve cell densities (if any) at different locations in the small intestine of horses, investigators selected two regions (Site #1 = midregion of the jejunum and Site #2 = mesenteric region of the jejunum) of this organ for study. Five equal sections of tissue were taken from each site and the average number of nerve cells for these five measurements are recorded in Table 1. The investigators would like to determine if nerve cell densities in these two regions are, on average, different. Let \( \alpha = 0.05 \).

[Notes: 1) Masty, J., (1983), Innervation of the Equine Small Intestine, Master’s Thesis, Purdue University

<table>
<thead>
<tr>
<th>Subject</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site #1</td>
<td>50.6</td>
<td>39.2</td>
<td>35.2</td>
<td>17.0</td>
<td>11.2</td>
<td>14.2</td>
<td>24.2</td>
<td>37.4</td>
<td>35.2</td>
</tr>
<tr>
<td>Site #2</td>
<td>38.0</td>
<td>18.6</td>
<td>23.2</td>
<td>19.0</td>
<td>6.60</td>
<td>16.4</td>
<td>14.4</td>
<td>37.6</td>
<td>24.4</td>
</tr>
<tr>
<td>d</td>
<td>12.6</td>
<td>20.6</td>
<td>12.0</td>
<td>-2.0</td>
<td>4.6</td>
<td>-2.2</td>
<td>9.8</td>
<td>-0.2</td>
<td>10.8</td>
</tr>
</tbody>
</table>

d = Site #1 – Site #2

1) Complete Table 1 by computing the differences \( d \) – in the direction specified and filling in the cells.

2) Inspect the QQ plot for the differences. Does it appear that this response variable comes from a distribution that is normal? Why/why not?

We have no reason to doubt that the data come from a normal distribution, the points follow (roughly) a linear pattern.

3) The investigators are concerned that the small sample size affected the ‘sensitivity’ of the A-D test for normality. Visual inspection of the QQ plot shows 2 distinct sections each with the same pattern of points. In addition, the distribution is bimodal (see histogram) so they have decided to take the conservative approach and use a non-parametric method. The technique to be used to complete this analysis is:

Wilcoxon Signed Rank Test
Wilcoxon Signed Rank Test: Nerve Cell Densities as Measured at Two Different Sites in the Small Intestine of Horses Example

4) The study null/alternative hypothesis pair is:

\[ \text{Ho: } M_d = 0 \]

\[ \text{Ha: } M_d \neq 0 \]

5) Desired \( \alpha = 0.05 \) type of test:

Left tailed | Right tailed | non-directional

6) Complete Table 2 by calculating the differences between each observed sample difference value and the hypothesized mean difference, ranking the absolute values of these differences (from lowest to highest), and then reintroducing the original sign of the difference.

Table 2: Differences - \( \mu_d \)

<table>
<thead>
<tr>
<th>Subject</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>12.6</td>
<td>20.6</td>
<td>12.0</td>
<td>-2.6</td>
<td>4.6</td>
<td>-2.2</td>
<td>9.8</td>
<td>-0.2</td>
<td>10.8</td>
</tr>
<tr>
<td>( \mu_d )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( d - \mu_d )</td>
<td>12.6</td>
<td>20.6</td>
<td>12.0</td>
<td>-2.6</td>
<td>4.6</td>
<td>-2.2</td>
<td>9.8</td>
<td>-0.2</td>
<td>10.8</td>
</tr>
<tr>
<td>Rank of (</td>
<td>d - \mu_d</td>
<td>)</td>
<td>8</td>
<td>9</td>
<td>7</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Sign of ( d - \mu_d )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Signed Rank</td>
<td>8</td>
<td>9</td>
<td>7</td>
<td>-2</td>
<td>4</td>
<td>-3</td>
<td>5</td>
<td>-1</td>
<td>6</td>
</tr>
</tbody>
</table>

7A) Because this is a nonparametric technique the desired significance level will not, generally, be achieved. The actual significance level is the alpha value that comes closest to the one that the investigators desire. For this study the actual significance level is:

\[ 0.039 \] (closest we can get to 0.05 without going over)

7B) At the specified level of significance, the critical signed rank sum value \( W_{crit} \) is:

\[ W_c = 40 \]

7C) Let \( W_s \) represent the test statistic for this technique. Based on your answer to problem 7B: State the decision rule:

\[ W_s = 8 + 9 + 7 + 4 + 5 + 6 = 39 \]

reject \( H_0 \) if \( W_s \geq W_c \)
Wilcoxon Signed Rank Test: Nerve Cell Densities as Measured at Two Different Sites in the Small Intestine of Horses Example

8) The test statistic - \( W_s \) - for this technique is called the Wilcoxon signed rank sum or, simply, the signed rank sum. It is the maximum of \( W_+ \) or \( |W_-| \):

\[
W_+ = 8 + 9 + 7 + 4 + 5 + 6 = 39
\]

\[
|W_-| = |-2| + |-3| + |-1| = 6
\]

\[
W_s = \max\{39, 6\} = 39
\]

9) The decision is: \( W_s = 39 < W_c = 40 \)  

10) The English interpretation is (be sure that the conclusion includes information about the actual achieved significance level):

At the .05 level, we don't have enough evidence to conclude that the nerve cell densities are different in the two sections of the small intestines.

11) Compare the results from Problem 10 with the paired t-test results from Output 2:

A) Are the results the same? Why/why not

The t-test leads to rejection aka diff. conclusion

\( \Rightarrow \) this is because the t-test is more powerful than the Wilcoxon

B) Which result should we believe?

- if we're confident in the normal assumption, then we believe t-test
- if we're not, then we go by Wilcoxon

\[ W_c = 37 \]

\( W_s = 39 \)

\( \Rightarrow \) reject \( H_0 \)

Note: if the hypotheses had been

\[ H_0: M_d \leq 0 \]

\[ H_a: M_d > 0 \]

would have had a critical value of

\( W_c = 37 \)

\( (W_s = 39) \)

\( \Rightarrow \) reject \( H_0 \)
Wilcoxon Signed Rank Test: Nerve Cell Densities as Measured at Two Different Sites in the Small Intestine of Horses Example

Figure 1: QQ plot of the differences

Figure 2: Histogram of the differences

Figure 3: Boxplot of the differences
Wilcoxon Signed Rank Test: Nerve Cell Densities as Measured at Two Different Sites in the Small Intestine of Horses Example

Output 1: Descriptive statistics of the differences

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>SE Mean</th>
<th>StDev</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>diff</td>
<td>9</td>
<td>7.33</td>
<td>2.60</td>
<td>7.79</td>
<td>-2.20</td>
<td>-1.10</td>
<td>9.80</td>
<td>12.30</td>
<td>20.60</td>
</tr>
</tbody>
</table>

Output 2: Paired t-test results for the differences

<table>
<thead>
<tr>
<th>Test of mu = 0 vs not = 0</th>
<th>test Stat</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>N Mean</td>
<td>StDev</td>
</tr>
<tr>
<td>diff</td>
<td>9 7.33</td>
<td>7.79</td>
</tr>
</tbody>
</table>

⇒ reject H₀

Output 3: Wilcoxon signed rank test results

Wilcoxon Signed Rank Test: diff

<table>
<thead>
<tr>
<th>Test of median = 0.000000 versus median not = 0.000000</th>
</tr>
</thead>
<tbody>
<tr>
<td>N for Wilcoxon Estimated</td>
</tr>
<tr>
<td>----------------------------</td>
</tr>
<tr>
<td>diff</td>
</tr>
</tbody>
</table>