Hypothesis Test – Means: Wing Area of Monarch Butterflies

Use the information in the following setting to answer questions 1 - 7:
Hypothesis testing: The rejection region approach -

As part of a larger study designed to characterize the body features of monarch butterflies that are found in California researchers captured many specimens from Oceano Dunes State Park. One of the variables of interest was the wing area. The data in Table 1 below are the results from 14 male specimens. Output 1 reports the descriptive statistics for the wing area data. The investigators would like to estimate the average wing area of all male monarchs in California during the season that the study was completed. Use a significance level of 0.05.


Table 1: Monarch Butterfly Wing Areas (cm²)

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>33.9</td>
<td>33.0</td>
<td>30.6</td>
<td>36.6</td>
<td>36.5</td>
<td>34.0</td>
<td>36.1</td>
</tr>
<tr>
<td>32.0</td>
<td>28.0</td>
<td>32.0</td>
<td>32.2</td>
<td>32.2</td>
<td>32.3</td>
<td>30.0</td>
</tr>
</tbody>
</table>

Output 1: Descriptive statistics for the monarch butterfly wing area data

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>SE Mean</th>
<th>StDev</th>
<th>Minimum</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>wing</td>
<td>14</td>
<td>32.814</td>
<td>0.662</td>
<td>2.476</td>
<td>28.000</td>
<td>31.650</td>
<td>32.250</td>
<td>34.525</td>
<td>36.600</td>
</tr>
</tbody>
</table>

1) Perform the hypothesis test: to check, if avg wing area is 35 cm²

Step 1

\[H_0: \mu = 35\]
\[H_a: \mu \neq 35 \text{ cm}^2\]

Step 2

\[\alpha = 0.05\]

Step 3

- random sample ✔
- indep. observations ✔
\[n = 14 \neq 30\]

Continue with the rest of the hypothesis test even if the assumptions do not hold.
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Step 4
\[ t_{test} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{32.814 - 35}{0.662} = -3.30 \]

Step 5
\[ p-value = 2 \cdot P(t_{13} > | -3.30 |) = 2 \cdot P(t_{13} > 3.3) \]
actual p-value = .006 = 2 \cdot (.0005 , .005) = (.001 , .01)

Step 6
Reject \( H_0, (p-value < .01) < (\alpha = .05) \)

Step 7
We have enough evidence at the \( \alpha = 0.05 \) level to conclude that the true avg. wing area of male monarch butterflies found in California is not 35 cm².

Use the CI method to carry out this 2-tailed (non-directional) hypothesis test:

2) \( H_0: \ \mu = 35 \)

\( H_a: \ \mu \neq 35 \)

3) Conf. Level = \( .95 \) \( \% \) \( df = 13 \) \( t_{crit} = 2.16 \)
\[ 100(1 - \alpha) \]
\[ t_* \]
4) The CI is: Recall the CI formula for μ

\[ \mu = \bar{x} \pm t \cdot \frac{s}{\sqrt{n}} \]

Or

\[ \mu = \bar{x} \pm SE \]

Output 2: CI and hypothesis test results for the wing area study

**One-Sample T: wing**

Test of \( \mu = 35 \) vs not = 35

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE</th>
<th>Mean 95% CI</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>wing</td>
<td>14</td>
<td>32.814</td>
<td>2.476</td>
<td>0.662</td>
<td>(31.385, 34.244)</td>
<td>-3.30</td>
<td>0.006</td>
</tr>
</tbody>
</table>

\[ CI = 32.814 \pm 2.16 \left( \frac{2.476}{\sqrt{14}} \right) = (31.384, 34.244) \]

5) Draw the CI diagram corresponding to these results

![CI Diagram]

6) The statistical decision is:

Reject \( H_0 \), as 35 is not contained.

7) The corresponding English interpretation is:

We are 95% confident that the true "μ" is between 31.384 and 34.244 cm\(^2\). Thus, the wing area is not 35 cm\(^2\).