Max-stable processes and annual maximum snow depth

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Outline

Motivation

Max-stable process - Schlather model

Annual maximum snow depth in Switzerland

Conclusion and outlook
SLF research

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- Avalanche Warning and Prevention
- Snow Physics, Permafrost and Climatology
- Avalanches, Debris Flows and Rockfall
- Mountain Hydrology and Torrents
- Ecosystem Boundaries
Extreme snowfall danger

- Avalanche warning and prevention
- Settlement and infrastructure management
- Floods and landslides caused by snowmelt
Our goal

- Try to understand how extreme snow depth is distributed over Switzerland
- Get return levels maps
- Simulate extreme snow depth
- ...

* A spatial model is required! *
Max-stable process - Schlather model

- $Z(x)$ process of block maxima (ex: maximum annual snow depth) at location $x$, assumed to be unit Fréchet

- Schlather model for the bivariate CDF in locations $x_1$ and $x_2$:

$$
P[Z(x_1) \leq z_1, Z(x_2) \leq z_2] = \exp \left[ -\frac{1}{2} \left( \frac{1}{z_1} + \frac{1}{z_2} \right) \left( 1 + \sqrt{1 - 2(\rho(h) + 1)\frac{z_1 z_2}{(z_1 + z_2)^2}} \right) \right]$$

**Covariance function** $\rho(h)$: how decreases the dependence between two locations when the distance increases.

$\rho(h) \in [0, 1]$, $\rho(h) \searrow 0$ when $h \nearrow$ (ex: Cauchy, ...)

**Extremal coefficient** $\theta(h)$:

$$\theta(h) = 1 + \sqrt{\frac{1 - \rho(h)}{2}} \in [1, 1 + \sqrt{0.5}] \approx [1, 1.7]$$

$\Rightarrow$ Complete independence ($\theta = 2$) is **never achieved**
Simulation of max-stable processes

Covariance function

Extremal index

Max-stable random field

Covariance function

Extremal index

Max-stable random field

Covariance function

Extremal index

Max-stable random field
Sensibility of Fréchet transform

- If \( Y(x) \sim GEV(\mu(x), \sigma(x), \xi(x)) \) then
  \[
  Z(x) = \left[ 1 + \xi(x) \frac{Y(x) - \mu(x)}{\sigma(x)} \right]^{\frac{1}{\xi(x)}} \sim \text{Fréchet}(1)
  \]

- Schlather model gives the bivariate distribution of \( Z(x_1) \) and \( Z(x_2) \)

- Approximation of the full likelihood by the pairwise likelihood (\textit{SpatialExtremes} package)

The covariance estimation is sensible to errors on the GEV parameters.

\textit{A (very) good GEV model is needed!}
Simulation: influence of noise in covariance estimation

\[ \mu(x) = \mu^0(x) + N(0, \sigma = 5) \]
Simulation: influence of noise in covariance estimation

\[ \mu(x) = \mu^0(x) + N(0, \sigma = 10) \]
Simulation: influence of noise in covariance estimation

\[ \mu(x) = \mu^0(x) + \mathcal{N}(0, \sigma = 15) \]
Data

- 130 stations in the Alps
- 43 winters (1966-2008)
- 4 regions: Plateau, North slope/South slope, Tessin
- 17 stations are removed from the analysis for validation
GEV model: Location

Residuals

Kriging residuals
GEV model: Scale

Residuals

Kriging residuals
GEV model: Shape

Residuals

Kriging residuals
GEV model: Validation

- **Predicted Location**
  - MLE Location vs. Predicted Location
  - Fitting: ●
  - Validation: ○

- **Predicted Scale**
  - MLE Scale vs. Predicted Scale
  - Fitting: ●
  - Validation: ○

- **Predicted Shape**
  - MLE Shape vs. Predicted Shape
  - Fitting: ●
  - Validation: ○
Schlather model estimation

\[ P \left[ Z(x_1) \leq z_1, Z(x_2) \leq z_2 \right] \]

\[ = \exp \left[ -\frac{1}{2} \left( \frac{1}{z_1} + \frac{1}{z_2} \right) \left( 1 + \sqrt{1 - 2(\rho(h) + 1)\frac{z_1 z_2}{(z_1 + z_2)^2}} \right) \right] \]

\( h \): distance between \( x_1 \) and \( x_2 \)

“Climatic” space:

\( h = \sqrt{\Delta x^T A \Delta x} \)

where

\[ A = \begin{pmatrix} \cos \phi & r \sin \phi & 0 \\ -r \sin \phi & \cos \phi & 0 \\ 0 & 0 & s \end{pmatrix} \]

\[ x = \begin{pmatrix} \text{longitude} \\ \text{latitude} \\ \text{altitude} \end{pmatrix} \]

→ extremal index \( \theta = c \) is an ellisoid
Extremal index in the Alps

North Slope of the Alps:
Pract. range = 750km in “x”
= 150km in “y” = 2000m in z
Extremal coefficient, dz=0

South Slope of the Alps
Pract. range = 620km in “x”
= 120km in “y” = 2800m in z
Extremal coefficient, dz=0

+ Simulations, conditional maps, ...
Conclusion

- Max-stable model for extreme snow depth
- Requires a good GEV model
- Illustration in Switzerland

Outlook:

- Introduce topography in covariance function
- Covariate for the GEV????