

Chapter 7

Probability

1

Random Circumstances

- A **Random circumstance** is one in which the outcome is unpredictable.

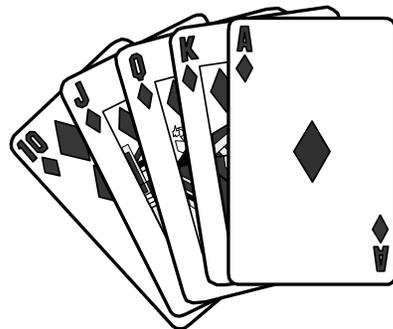
My history professor used to give out a list of 10 possible questions for his exams. He would choose 4 from his list of 10.

Naturally, we didn't know which questions he would use for the exam so we had to either prepare for all 10 or risk being given a problem we hadn't studied

Chapter 7 # 2

Probability

- Probability is the chance that a specific event actually occurs.
- Probabilities are nothing more than proportions
- Probabilities are ALWAYS numbers between 0 and 1



Chapter 7 # 3

Interpretations of Probability (1)

The Relative Frequency Interpretation

The probability of a specific outcome is **the proportion of times the outcome would occur over the long run**

Chapter 7 # 4

Probability of Male versus Female Births

Long-run relative frequency of males born in the United States is about .512.

Information Please Almanac (1991, p. 815).

TABLE 7.1 ■ Relative Frequency of Male Births over Time

Weeks of Watching	Total Births	Total Boys	Proportion of Boys
1	30	19	.633
4	116	68	.586
13	317	172	.543
26	623	383	.615
39	919	483	.526
52	1237	639	.517

Chapter 7 # 5

Probability of lost luggage

“1 in 176 passengers on U.S. airline carriers will temporarily lose their luggage.”

This number is based on data collected over the long run. So the **probability** that a randomly selected passenger on a U.S. carrier will temporarily lose luggage is 1/176 or about .006.

Chapter 7 # 6

Expressing relative frequency

- The proportion of passengers who lose their luggage is 1/176 or about .006.
- About 0.6% of passengers lose their luggage.
- The probability that a randomly selected passenger will lose his/her luggage is about .006.
- If 100,000 passengers pass through DIA in a weekend, you expect 600 of them to experience lost luggage.

Chapter 7 # 7

The Personal Probability Interpretation

The **personal probability** of an event is the degree to which an individual believes the event will happen.

Sometimes **subjective probability** used because the degree of belief may be different for each individual.

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Probability terminology

- **Sample space**: the list of all possible outcomes of a random circumstance.
- **Simple event**: one possible outcome in the sample space
- **Complex Event**: a collection of more than one simple events in the sample space

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Probability terminology - example

Experiment: Draw a single card from a fair deck

- **Sample space**: Each of the 52 possible outcomes
- **Simple event**: Draw a king
- **Complex Event**: Draw a red king

Chapter 7 # 10

Probability relationships

Conditions for Valid Probabilities

1. Each probability is between 0 and 1.
2. The sum of the probabilities over all possible simple events is 1.

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Probability terminology

The **complement** of an event is whatever is left over in the sample space after removing that outcome

Notation: A^C represents the **complement** of event A.

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Probability terminology

Draw a single card from a fair deck

- **Sample space**: Each of the 52 possible outcomes
- **Simple event**: Draw a king
- **Complement of Event**: Draw anything but a king
- **Probability of the Simple event**: $\Pr\{K\} = 4/52$
- **Probability of Complement**: $\Pr\{K^c\} = 48/52$

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Probability terminology

Two events are **mutually exclusive**, or **disjoint**, if they cannot occur together

Example: Randomly select an adult who is between 55 and 65 years old.

Event A: Randomly selected individual is a woman

Event B: Randomly selected individual has prostate cancer

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Probability terminology

Two events are not **mutually exclusive** if they can occur together

Example: Randomly select an adult who is between 55 and 65 years old.

Event A: Randomly selected individual is a woman

Event B: Randomly selected individual has borne at least one child

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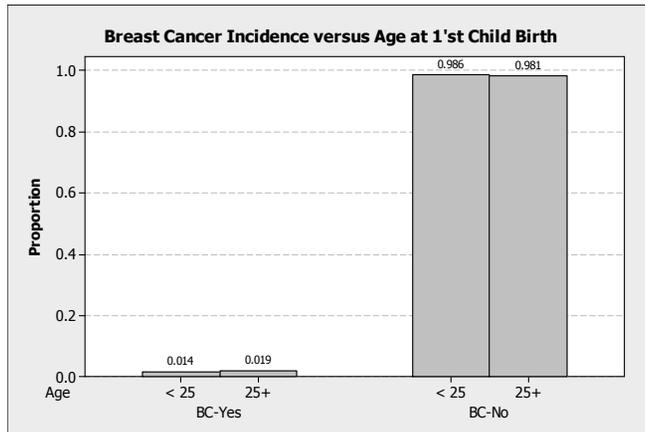
Probability terminology

Two events are **independent** if the occurrence of one of the events does not change the probability that the other will/has occurred

Two events are **dependent** if the occurrence of one of the events does changes the probability that the other will/has occurred

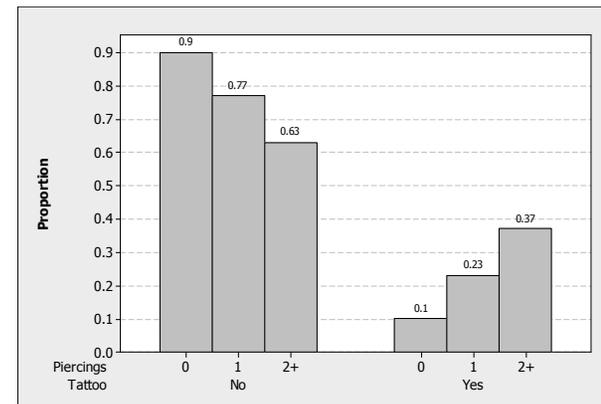
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Age at first child's birth and probability of developing breast cancer are independent because her chances of developing breast cancer are (very nearly) the same for all age categories



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Number of ear piercings and presence of a tattoo are not independent because there is a higher likelihood for a man to have a tattoo if he has two or more piercings than none



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Probability terminology

The **conditional probability** is the long-run relative frequency with which a specified event occurs when another event has, also, occurred.

It is written as $\Pr\{\text{Event 1} \mid \text{Event 2}\}$

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Example of conditional probability

A survey of 78,564 high school students resulted in the following:

- 1) 39,989 were boys
- 2) 38,575 were girls
- 3) 9,157 of the boys gambled at least once a week
- 4) 1,736 of the girls gambled at least once a week

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The Contingency Table

Gender	Yes Gamble	No Gamble	Totals
Boy	9157	30832	39989
Girl	1736	36839	38575
Totals	10893	67671	78564

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Example of conditional probability

$$\Pr\{\text{gamble} \mid \text{boy}\} = \frac{9157}{39989} = 0.229$$

$$\Pr\{\text{gamble} \mid \text{girl}\} = \frac{1736}{38575} = 0.045$$

Since the chances of weekly gambling are quite different for each category of gender the variables are not independent

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