

## Normal Probabilities Practice Problems Solution

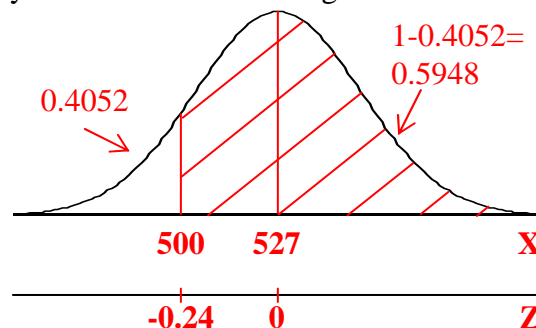
1. Most graduate schools of business require applicants for admission to take the Graduate Management Admission Council's GMAT examination. Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112. What is the probability of an individual scoring above 500 on the GMAT?

Normal Distribution  $Z = \frac{500 - 527}{112} = -0.24107$

$\mu = 527$

$\sigma = 112$

$\Pr\{X > 500\} = \Pr\{Z > -0.24\} = 1 - 0.4052 = \boxed{0.5948}$



2. How high must an individual score on the GMAT in order to score in the highest 5%?

Normal Distribution

$\mu = 527$

$\sigma = 112$

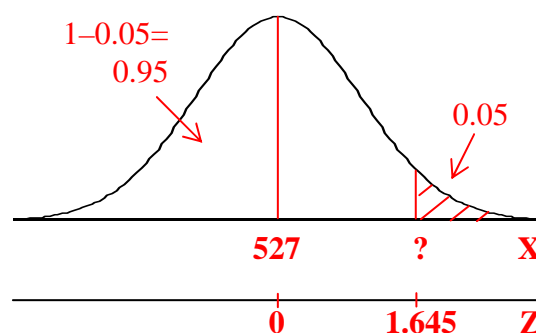
$P(X > ?) = 0.05 \Rightarrow P(Z > ?) = 0.05$

$P(Z < ?) = 1 - 0.05 = 0.95 \Rightarrow Z = 1.645$

$X = 527 + 1.645(112)$

$X = 527 + 184.24$

$X = \boxed{711.24}$



3. The length of human pregnancies from conception to birth approximates a normal distribution with a mean of 266 days and a standard deviation of 16 days. What proportion of all pregnancies will last between 240 and 270 days (roughly between 8 and 9 months)?

Normal Distribution  $Z = \frac{240 - 266}{16} = -1.625$

$\mu = 266$

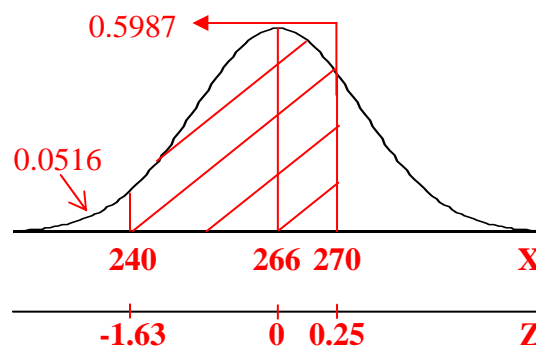
$Z = \frac{270 - 266}{16} = 0.25$

$\sigma = 16$

$P(240 < X < 270) = P(-1.63 < Z < 0.25)$

$P(-1.63 < Z < 0.25) = P(Z < 0.25) - P(Z < -1.63)$

$P(-1.63 < Z < 0.25) = 0.5987 - 0.0516 = \boxed{0.5471}$



4. What length of time marks the shortest 70% of all pregnancies?

Normal Distribution

$\mu = 266$

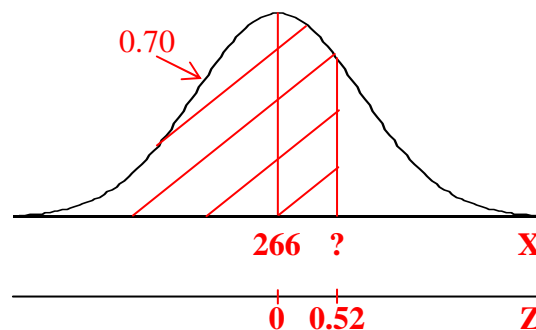
$\sigma = 16$

$P(X < ?) = 0.70 \Rightarrow P(Z < ?) = 0.70 \Rightarrow Z = 0.52$

$X = 266 + 0.52(16)$

$X = 266 + 8.32$

$X = \boxed{274.32}$



## Normal Probabilities Practice Problems Solution

5. The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given year?

**Normal Distribution**

$$Z = \frac{2500 - 4300}{750} = -2.40$$

$$Z = \frac{4200 - 4300}{750} = -0.13333$$

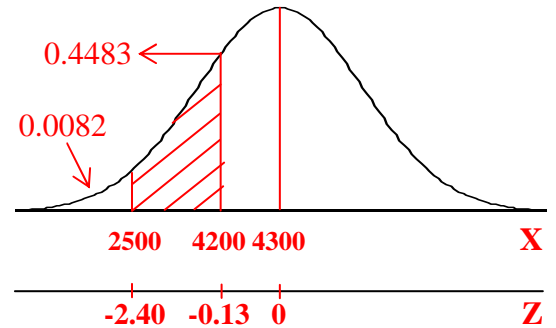
$$\mu = 4300$$

$$\sigma = 750$$

$$P(2500 < X < 4200) = P(-2.40 < Z < -0.13)$$

$$P(-2.40 < Z < -0.13) = P(Z < -0.13) - P(Z < -2.40)$$

$$P(-2.40 < Z < -0.13) = 0.4483 - 0.0082 = \boxed{0.4401}$$



6. What number of burnt acres corresponds to the 38<sup>th</sup> percentile?

**Normal Distribution**

$$\mu = 4300$$

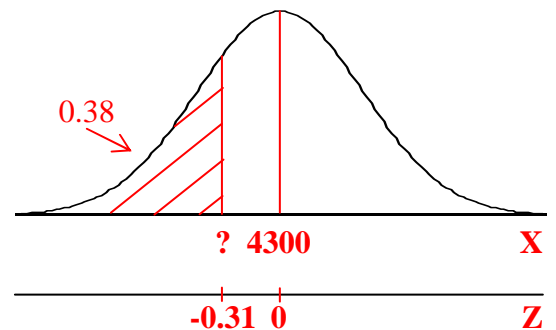
$$\sigma = 750$$

$$P(X < ?) = 0.38 \Rightarrow P(Z < ?) = 0.38 \Rightarrow Z = -0.31$$

$$X = 4300 + (-0.31)(750)$$

$$X = 4300 - 232.5$$

$$X = \boxed{4067.5}$$



7. The Edwards's Theater chain has studied its movie customers to determine how much money they spend on concessions. The study revealed that the spending distribution is approximately normally distributed with a mean of \$4.11 and a standard deviation of \$1.37. What percentage of customers will spend less than \$3.00 on concessions?

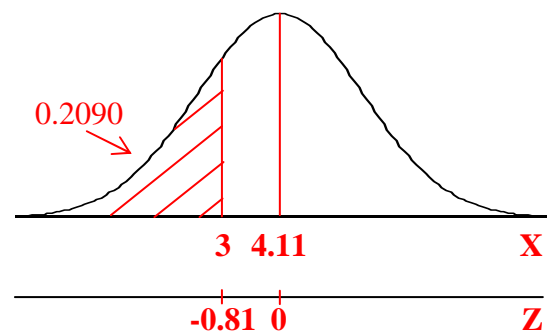
**Normal Distribution**

$$Z = \frac{3.00 - 4.11}{1.37} = -0.81021$$

$$\mu = 4.11$$

$$\sigma = 1.37$$

$$P(X < 3.00) = P(Z < -0.81) = 0.2090 \Rightarrow \boxed{20.9\%}$$



8. What spending amount corresponds to the top 87<sup>th</sup> percentile?

**Normal Distribution**

$$\mu = 4.11$$

$$\sigma = 1.37$$

$$P(X > ?) = 0.87 \Rightarrow P(Z > ?) = 0.87$$

$$P(Z > ?) = 0.87 \Rightarrow P(Z < ?) = 1 - 0.87 = 0.13 \Rightarrow Z = -1.13$$

$$X = 4.11 + (-1.13)(1.37)$$

$$X = 4.11 - 1.5481$$

$$X = 2.5619$$

$$X = \boxed{\$2.56}$$

