PREDICTING COLORADO RIVER SANDBAR SIZE USING GLEN CANYON DAM RELEASE CHARACTERISTICS

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ABSTRACT

The purpose of this study was to determine the impact of water releases from the Glen Canyon Dam (GCD) upon the sandbars downriver in the Colorado River using aerial photography data collected for 58 sandbars in 1990-1. In the three models considered here, the response is net change in sandbar size averaged over the sandbars measured on each flight.

In this paper we describe three models. Two regression models for correlated data predict average net change in sandbar size per flight for sandbars below the Little Colorado River. In these models, mean daily water discharge from the dam and presence or absence of sand added to the river from the Little Colorado River were significant predictors.

Due to limitations of these models related to the use of small sample sizes to estimate parameters in a model for correlated data, we also consider a standard regression model to predict net change per flight averaged over *all* sandbars included in the study. For the regression model, mean daily water discharge from the dam and the average daily maximum increase in discharge level (upramp) were significant predictors of average net change per flight.

The regression model indicates that as mean daily discharge increases and upramp remains fixed, the average net change of sandbars along the river increases (in other words, the sandbars tend to increase in size on average). As mean daily maximum upramp increases and mean daily discharge remains fixed, the average net change of sandbars along the river decreases (in other words, the sandbars tend to decrease in size on average).

In this report we also provide recommendations for future studies. We suggest that designers of future studies should consider systematic stratified sampling. This may reduce study costs and should result in improved estimates of the parameters of interest. We also suggest that smaller sampling intervals be used in future studies. Smaller sampling intervals would allow for a better understanding of the natural processes and responses to dam operations. Since smaller sampling intervals might increase study costs, we suggest more frequent observations over *fewer* sandbars to increase the amount of information gained about the problem of interest.

List of Descriptors/Identifiers

GCD: Glen Canyon Dam

GCES: Glen Canyon Environmental Studies (office of the BOR)

LCR: Little Colorado River

NPS: National Park Service

Q_s: Sediment transport capacity

Q_w: Mean daily discharge

BOR: U.S. Bureau of Reclamation

USGS: U.S. Geological Survey

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Chapter 1

INTRODUCTION

The purpose of this study is to determine the impact of water released from the Glen Canyon Dam (GCD) on sandbars downriver in the Colorado River (Figure 1). The GCD was constructed on the Colorado River between 1950 and 1963. Besides being a very efficient sediment trap, it is an important hydropower generator designed to meet peak loads in the Intermountain and Western United States.

Due to the reduction in the amount of sediment being carried by the Colorado river, many of the sandbars are decreasing in size (Kearsley *et al.*, 1994). The low sediment supply from Lake Powell to the Colorado River is not capable of replenishing the sandbars, causing the river to erode the sandbars downriver. The dam also prevents the annual flood cycle which naturally rejuvenates sandbars each year. The sandbar erosion has had a widespread impact upon the environment, affecting many species of birds (Brown and Johnson 1987) and fish (Maddux *et al.* 1987) along the river and also affecting the recreationalists who use the Colorado River (Kearsley *et al.*, 1994).

The data described here were collected from September 1990 through July 1991 during the Glen Canyon Environmental Studies (GCES) phase II test flows. For 17 testing periods, a test flow (a pattern of water release) was released from the dam for eleven days and then sandbar sizes were recorded for sandbars downriver from the GCD.

The original goals of the project described in this report were described in the original proposal as follows. " The aerial photography data base will be analyzed using robust statistical procedures in order to develop a model or models that predict sandbar size from various hydrologic, geomorphic and geographic parameters. An additional goal is to use statistical analyses to estimate the most efficient sample size for monitoring sandbars over long and short time periods and with respect to changes in flow patterns."

Our original goal was to produce a space/time model to predict sandbar sizes based on sandbar characteristics and dam release measurements. The model was hoped to provide managers and scientists with some guidelines on how different patterns of dam water releases impacted different types of sandbars. Preliminary data analysis along with a survey of available statistical methodology indicated that the large amount of missing data made this goal unattainable. This was disappointing because this is the largest data set ever obtained for a sample of Grand Canyon sandbars; indeed, a large sample of sandbars was frequently monitored over a relatively long period of time under controlled dam releases. By averaging over the net change in sandbar size of all sandbars per flight, we reduced the problem to one involving only correlations between observations over time instead of correlations over both space and time. This prevents us from making inferences about different *types* of sandbars or specific reaches of the river, but still allows us to investigate the relationship between water release patterns and sandbar size.

In this paper we develop three models to predict average net change in sandbar size. The first two models are regression models for correlated data. The best predictors for average net change of sandbars downstream of the Little Colorado River (LCR) are the average daily water discharge from the dam and the presence or absence of sediment input from the LCR. The previous three observations of average net change, weighted by the number of days between observations are the auto-regressive predictors for average net change in sandbars below the LCR.

Due to limitations in these two regression models for correlated data related to the use of small sample sizes to estimate parameters in a model for correlated data, a regression model to predict the average net change in sandbars size over the entire Colorado River is also introduced. The predictors for this model are the average daily water discharge from the dam and the average increase in amount of discharge along the river (upramp). This model indicates that as mean daily discharge increases and upramp remains fixed, the average net change of sandbars along the river increases (in other words, the sandbars tend to increase in size on average). As mean daily maximum upramp increases and mean daily discharge remains fixed, the average net change of sandbars along the river decreases (in other words, the sandbars tend to decrease in size on average).

In addition to introducing these models in this report, we also provide suggestions for future studies. In Chapter 5 we provide several suggestions for future sampling plans and discuss other aspects that are crucial to the design of future studies.

With the help of the results from this paper, the NPS and the U.S. Bureau of Reclamation (BOR), which runs the dam, may be able work together to control the average net change of sandbar size and possibly rebuild some of the sandbars with flows in the range of hydropower production. At the very least, NPS and BOR will be able to better interpret trends in data from past monitoring programs and improve designs of new monitoring programs.

Overview

In Chapter 2 , we discuss the data collection process and describe the data collected for this study. In Chapter 3, the two regression models for correlated data for sandbars below the LCR are discussed and the regression model for all sandbars along the Colorado River is also described. Chapter 4 provides conclusions for this report. Chapter 5 includes suggestions for future studies and other management considerations. Appendix A describes the technical details of the models discussed in Chapter 3. In Appendix B we provide the final project budget, and Appendix C describes and reports the data used for the analyses described in this report. A diskette including the project data is also included with this report.

Figure 1.1: Map of Study Area. Approximate Location of Map

Chapter 2

METHODS

2.1 Dam Releases

During the period the data in this report were obtained, the typical minimum discharge from the Glen Canyon Dam was approximately 85 m^3/s (3000 cubic feet per second, cfs) and the maximum is approximately 800 m^3/s (28,000 cfs). Normal operation of the dam results in a midday peak of water discharge coinciding with maximum demand for hydropower. For this study, test flows of varying dam flow patterns were run for a duration of 11 days. Some of these test flows included varying discharge levels throughout the eleven day period while others consisted of a constant discharge over the test flow period. Between test flows the dam released water at a constant rate of 142 m^3/s (5000 cfs) for three days. These "between test periods" were convenient evaluation periods for various environmental measurements.

2.2 Data Collection

Sandbar size measurements were collected via photographs taken during 17 helicopter flights from September 1990 to July 1991. The same 58 sandbars out of a population of about 600 sandbars along approximately 390 kilometers (230 miles) of river below the GCD were photographed on each flight. The helicopter flights were taken during evaluation periods so the river would be at the same water level for all flights. Most flights were intended to be evenly spaced over time; however, due to weather conditions and other difficulties, some flights are unevenly spaced over time. The photographs were digitized into a perimeter file which was processed by a computer program that counted the number of pixels filling the perimeter. The collection and reduction of these data are described in Cluer 1995b. These results were used to estimate the size of each sandbar for each flight. Due to large amounts of missing data for two flights, we included data from only 15 of the 17 flight dates in our analyses.

2.3 Descriptive Statistics

2.3.1 "Per Flight" Predictors

Several variables were collected on each flight. These "per flight" predictors include date of the flight, the amount of time between flights, and several test flow characteristics. Table 2.1 reports the summary statistics for the number of days between flights and the test flow characteristics. Days between flights is simply the number of days between the current and previous flight. Mean daily discharge (Q_w) is the daily average discharge from the dam over a flight period. Upramp measures the increase in discharge level over time at a specified point on the river. Mean daily maximum upramp is the average of the maximum rise in discharge per day at 5 different gauging stations along the river. The upramp statistics reported here are averaged over each inter-flight period. Downramp measures the decrease in discharge level over time at a specified point on the river. Mean daily maximum downramp is the average of the maximum decreases in discharge per day at 5 different gauging stations along the river.

Two main tributaries along the study area have sediment gauging stations. One tributary, the Paria River is at river mile 0.8, and the other is the LCR, at mile 61.2 (Figure 1). Data supplied by the U.S. Geological Survey (USGS) include the total amount of sediment being added to the Colorado River by these tributaries. Approximately 17% of the total sediment supply is sand sized particles (Greg Fisk,

Table 2.1: Summary Statistics

Predictors	**Mean**		**Std. Dev.**	**Range**
Days Between Flights	19.9	(Days)	15.3	12 – 70
Mean Daily Discharge	336.0	(m^3/s)	83.3	194.2 – 491.2
Mean Daily Max Upramp	121.0	(m^3/s)	63.7	7.1 – 209.5
Mean Daily Max Downramp	117.3	(m^3/s)	62.5	12.2 – 236.2
Transport Capacity	22007.2	(tones)	18491.6	3000 – 61000
Sand Supply	506940	(tones)	1243616.0	0 – 4529839
Responses				
Total Area	4780.7	(m^2)	4790.5	457 – 27115
Fill Area	213.6	(m^2)	377.7	0 – 2979
Cut Area	-210.9	(m^2)	-359.7	-2512 – 0
Net Change	13.2	(m^2)	478.5	-2411 – 2900

USGS, personal communication). Because the NPS was interested primarily in how *sand* affects the sandbars, we used a fraction (17%) of the total sediment load as the estimate of sand supply.

The descriptive statistics reported for sand supply describe the sum of the daily sand load added to the river by the LCR during each interflight period. Because the LCR does not flow continuously, 6 out of 15 observations are zeroes which represent times when there was no sand supply being added to the Colorado River. Sediment Transport Capacity (Q_s), a measure of potential for sediment movement, is calculated from hourly water discharge measurements using the formula (Smillie, *et al.*, 1992)

$$Q_s = 4.6047^{-10} Q_w^{3.2228}. \tag{2.1}$$

Both sand supply and sediment transport capacity are measured in tones (metric tons).

When we consider the number of days between flights, three observations have large intervals between flights (Figure 2.1). Previous sandbar studies have typically involved longer intervals between observations as few flights or river trips are made per year to study the sandbars. While these three observations with large intervals between flights are outliers in this study, they are more representative of typical sandbar monitoring studies.

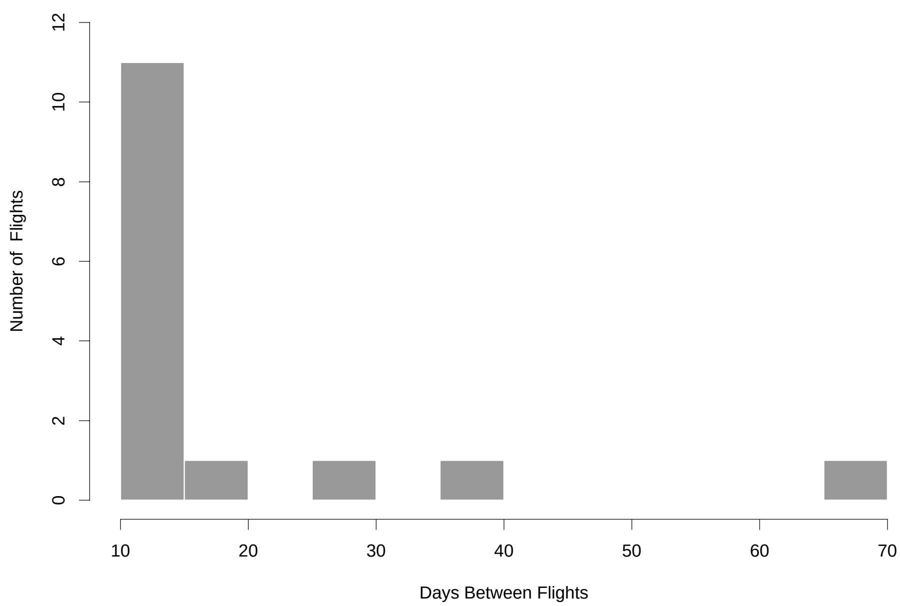

Figure 2.1: Histogram of Days Between Flights

2.3.2 "Per Sandbar" Predictors

Four variables, called "per sandbar" variables, characterize individual sandbars. The "per sandbar" predictors included river mile at which the sandbar was located, the bank of the river where the sandbar was located (left or right), the type of sandbar, and the size of the sandbar measured on each flight. There were four categories of sandbar types: upper pool, reattachment bar, separation bar, and margin bar. An upper pool bar is a sandbar formed just upriver from a debris flow or other channel constriction. A separation bar is one that is formed on the downriver lobe of a debris flow. A reattachment bar is a sandbar that is formed in the lee of a debris flow. This classification includes the eddy deposits found in the flow recirculation zone that forms in channel expansions. A margin bar is a general term for any deposit along the bank. It is important to note that a sandbar can include combinations of the above categories.

Four size characteristics were recorded for each sandbar for each flight, including gross area, net change in size, area of erosion, and area of deposition. Net change is the net change in area between flights for each sandbar. Net change is also the response of primary interest. In the models described below, we use net change as the response because it allows for both increasing and decreasing sandbar sizes. The formula for net change for flight i is

$$Net_i = Cut_i + Fill_i. \tag{2.2}$$

Figure 2.2 shows the mean net change for all sandbars observed on each flight.

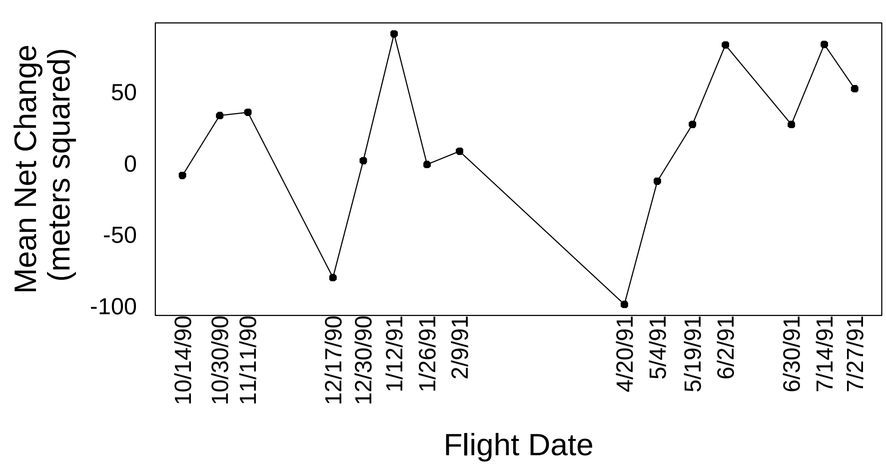

Figure 2.2: Mean Net Change (m^2) by Flight Date.

Table 2.2: Sandbar Summary Statistics

Predictor	**Percentage**
Bank	
right	53.5%
left	46.6%
Bar Type	
Margin Bars	10.3%
Reattachment Bars	58.6%
Separation Bars	41.4%
Upper Pool Bars	27.6%

2.4 Missing Data

There was considerable missing data for this study. Of 986 possible observations of sandbar size (58 sandbars $\times$ 17 flights), only 692 were available for analysis. Data were missing for various reasons including some sandbars that were missed by the photographer during a flight. The primary reason for missing data, however, was blurry photographs due to poor lighting and helicopter vibration due to low flying speeds. Figure 2.3 shows that there are more missing sandbars for the early flights in the study. Table 2.3 lists the flights that are missing for each sandbar. The number of missing observations is fairly constant along the river (Figure 2.4).

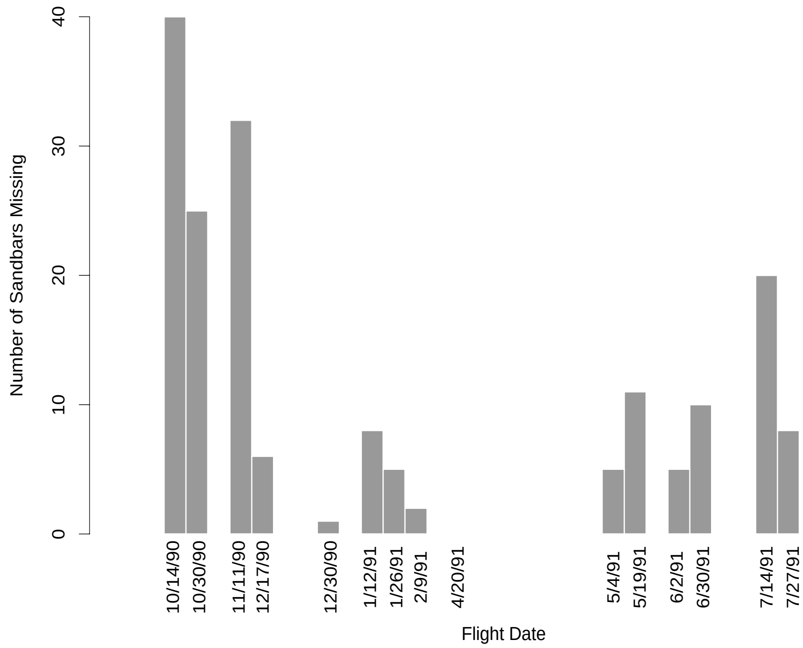

Figure 2.3: Histogram of the Number of Missing Sandbars for Each Flight. For the 4/20/91 flight there were no missing sandbars.

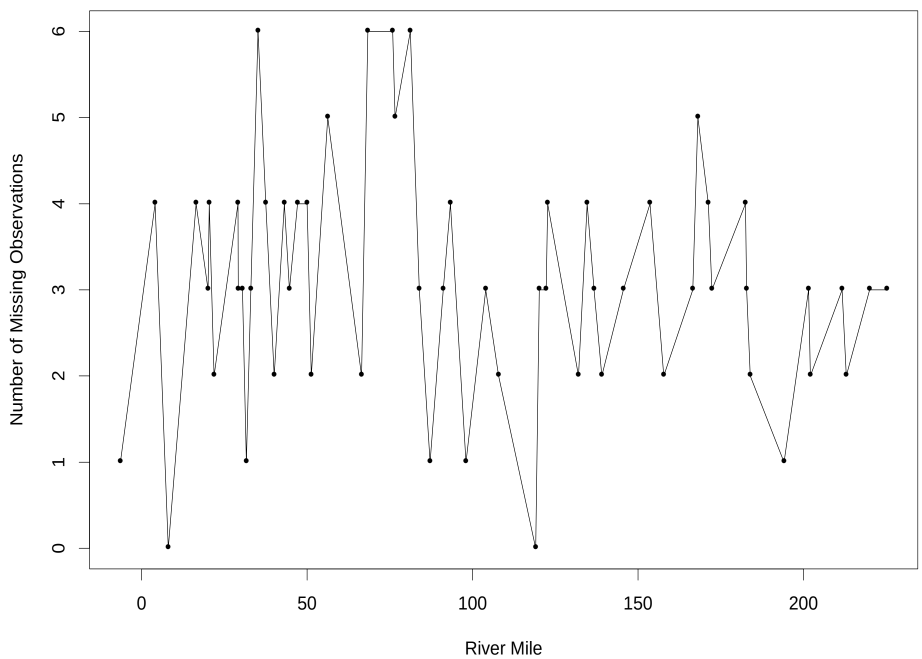

Figure 2.4: Number of Missing Observations for each Sandbar (River Mile)

Table 2.3: Table of Missing Observations

Site	River Mile	# of Missing Observations	Missing Flight (By Julian Day)
1	-6.5	1	286
2	4.0	4	272, 286, 314, 559
3	8.0	0	0
4	16.4	4	488, 503, 517, 559
5	20.0	3	302, 517, 559
6	20.4	4	272, 286, 302, 559
7	21.8	2	302, 363
8	29.1	4	272, 286, 302, 314
9	29.2	3	272, 302, 517
10	30.4	3	286, 314, 517
11	31.6	1	363
12	33.0	3	272, 350, 545
13	35.1	6	272, 286, 503, 517, 545, 559
14	37.5	4	272, 286, 302, 545
15	40.0	2	272, 545
16	43.1	4	302, 363, 474, 545
17	44.6	3	272, 286, 545
18	47.1	4	272, 286, 314, 517
19	50.0	4	286, 363, 488, 545
20	51.2	2	286, 363
21	56.2	5	272, 286, 302, 503, 545
22	66.4	2	286, 545
23	68.3	6	272, 286, 302, 314, 474, 545
24	75.8	6	272, 286, 302, 503, 545, 559
25	76.5	5	272, 286, 302, 517, 545
26	81.2	6	272, 302, 363, 376, 390, 545
27	83.9	3	302, 376, 488
28	87.1	1	488
29	91.1	3	272, 302, 363
30	93.3	4	286, 376, 517, 545

Table 2.4: Table of Missing Observations (Continued)

Site	River Mile	# of Missing Observations	Missing Flight (By Julian Day)
31	98.0	1	545
32	103.9	3	272, 488, 545
33	107.8	2	272, 545
34	119.0	0	0
35	120.1	3	272, 302, 559
36	122.2	3	272, 302, 474
37	122.7	4	272, 302, 474, 545
38	132.0	2	272, 302
39	134.6	4	272, 286, 302, 517
40	136.7	3	272, 286, 302
41	139.0	2	272, 302
42	145.5	3	272, 302, 474
43	153.6	4	272, 302, 363, 488
44	157.7	2	272, 302
45	166.5	3	272, 302, 559
46	168.0	5	272, 302, 376, 488, 545
47	171.2	4	272, 286, 302, 314
48	172.3	3	272, 302, 488
49	182.4	4	272, 286, 302, 488
50	182.8	3	272, 286, 302
51	183.9	2	272, 488
52	194.1	1	390
53	201.5	3	272, 286, 302
54	202.0	2	272, 286
55	211.6	3	272, 488, 545
56	212.9	2	376, 517
57	219.9	3	272, 286, 503
58	225.2	2	272, 286

Chapter 3

RESULTS AND DISCUSSION

Our original goal was to determine which sandbar characteristics and dam release measurements best predict net change in sandbar size for each flight. Since the response in this case would be net change in size for a given sandbar on a given flight, we originally sought a model which accounted for correlation between the responses over both space (river mile) and time (flight). Unfortunately, preliminary analyses along with a comprehensive survey of available statistical methodology indicated that the large amount of missing data and the unequally spaced time intervals between observations made this goal of a space/time model unattainable. This was disappointing because this is the largest data set ever obtained for a sample of Grand Canyon sandbars; indeed, a large sample of sandbars was monitored over a relatively long period of time.

To overcome these difficulties we sought a simplified model. Exploratory analyses of the non-missing data indicated no significant correlation between observations over space. In other words, net change in sandbar size for sandbars in close proximity were not more similar than observations for sandbars that were quite far apart. Therefore, we chose to average all observations collected per flight. This reduced the complexity of the problem and we now needed only to consider correlations over observations in time.

In this chapter we will discuss three different models to predict net change in sandbar size. For all three models discussed below, the response is the net change in sandbar size averaged over each flight; thus there is one response per flight and 15 responses in all. For the remainder of this report we will use the term "net change" to refer to average net change in sandbar size for each flight. Two models consider net change averaged over the 37 sandbars below the Little Colorado River. The final model considers net change averaged over *all* sandbars measured on each flight.

Preliminary analyses indicated relationship between observations over time for net change in sandbars below the LCR. The first two models discussed below are regression models for correlated data for sandbars below the LCR. A lag model and an auto-regressive model will be introduced, and the results for each model will be discussed. There are several limitations of these regression models for correlated data which will be discussed in Section 3.1.3. Due to the limitations of these models, the model we will focus on is a multiple regression model for net change for all of the sandbars in the study. This model will be described in Section 3.2.1.

3.1 Sandbars Below the Little Colorado River

Since little sediment flows through the GCD, the majority of sediment added to the Colorado River comes from its tributaries. One goal of this project was to determine whether net change is influenced by the amount of sediment supply from tributaries of the Colorado River. Both the Paria river and the LCR have sediment gauging stations which measure the amount of sand being added to the Colorado River by the tributaries (Figure 1).

During the study period, the LCR flowed more consistently than the Paria River. Of the 15 flight dates for this study, there were only 4 interflight periods during which sand was added to the Colorado River from the Paria River. There were 9 interflight periods during which sand was added from the LCR. Because it would be difficult to model the combined effects of the two rivers upon the sandbars of the Colorado River and also due to the small number of observations from the Paria River, we chose to focus on sand supplied by the LCR only. Since sand load from the LCR can only impact sandbars downstream of the confluence of the LCR and the Colorado Rivers, only those 37 sandbars downstream of the confluence were included in these analyses. Figure 3.1 suggests that net change tends to be positive when no sand is being added to the Colorado River and that net change is highly variable when sand *is* being added to the river. For the results that follow, the i^{th} response is the mean net change for the sandbars below the LCR on the i^{th} flight.

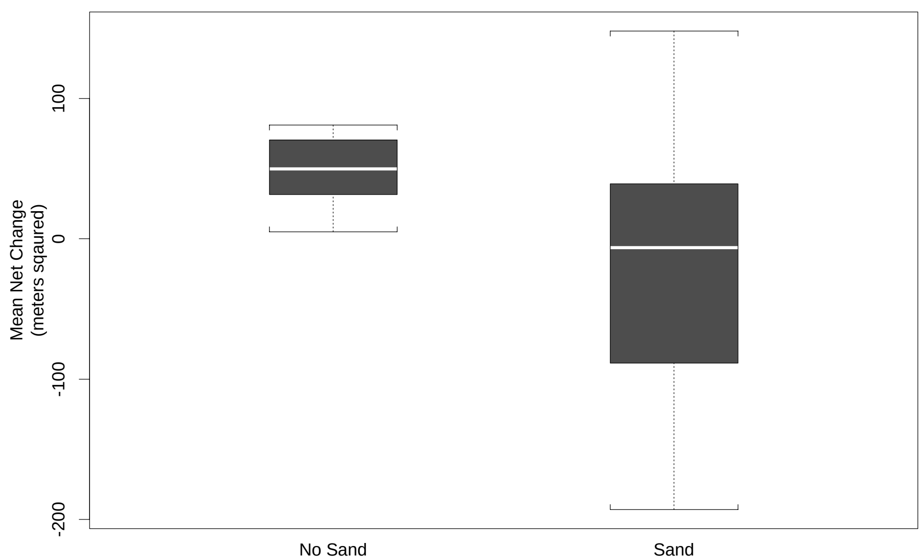

Figure 3.1: Boxplot of Mean Net Change (m^2) per Flight of Sandbars Below the Little Colorado River Versus Presence/Absence of Sand Supply. "No Sand" indicates that no sand was added to the river, and "Sand" indicates sand was added to the river.

An auto-correlation function describes the serial dependence between observations over time. Since the auto-correlation function assumes equally spaced time intervals between observations, we used the auto-correlation function for exploratory analyses only. The plot of the auto-correlation function (Figure 3.2) implies that a lag of three flights could be important in this model. In other words, net change from three flights ago may be a good predictor of current net change. Thus the antecedent conditions were important in predicting the response. We included a weighted average of the mean net change in sandbar size for the previous three flights.

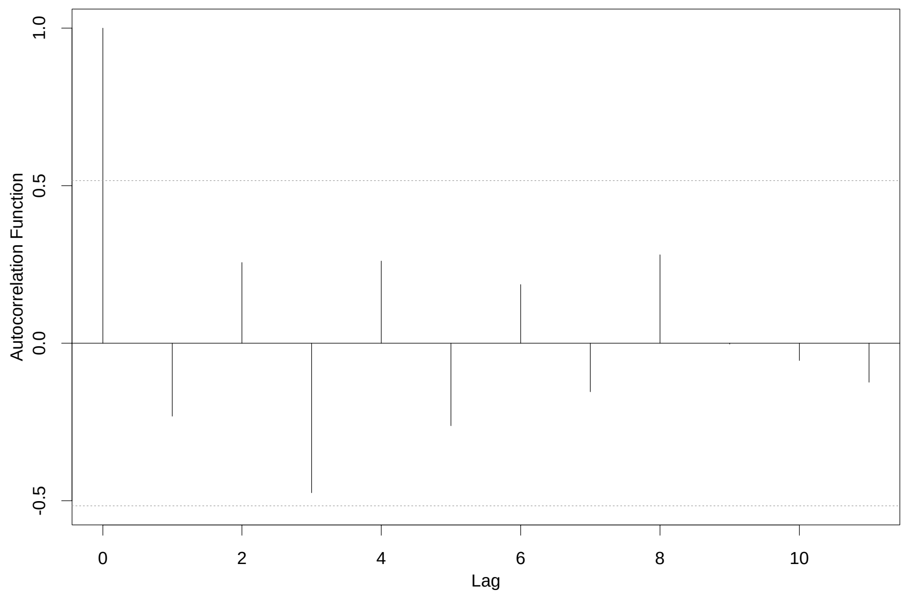

Figure 3.2: Autocorrelation Plot for Average Net Change for Sandbars Below the Little Colorado River. The dotted lines indicate approximate 95% confidence interval limits. This plot indicates that net change from three flights ago may be a good predictor of current net change.

Below we describe two models which predict net change in sandbar size. The lag model accounts for a lag in the response (a weighted average of the previous three flights), but does not adjust the predictors for the previous three flights. The second model, the auto-regressive model, accounts for the lag in the response *and* in the predictors. In both models the weighting in the lag term takes the unequally spaced flight time intervals into account. In addition, observations that occurred earlier in time are downweighted. The weighting structure is described further in the next section.

The lag and auto-regressive models described here are typically used to model correlation over space. In this paper, we use these models to model correlation over time. We chose these models because they allow for the variable number of days between flights and because they are relatively straightforward to understand. Limitations of the models are discussed in Section 3.1.3.

3.1.1 Lag Model

The lag model takes into account only a lagged response. We considered this model primarily for its simplicity. In this model net change, Y_i, is a function of predictors, X_i, and a weighted average of the net change for the previous three flights. The lag model is of the form:

$$Y = X\beta + \rho WY + \epsilon \text{ with } \epsilon \sim N(0, \sigma^2), \tag{3.1}$$

where $W = [\omega_{i,j}]$ where $\omega_{i,j}$ is a nonnegative weight which is representative of the 'degree of possible interaction' of observation i and j and $\omega_{ii} = 0$ (Upton and Fingleton, 1985). The W matrix accounts for the unequally spaced observations over time. In the lag model ρ can be interpreted as a measure of dependence between observations of the response. Inclusion of the WY term allows the other explanatory variables to be assessed after accounting for the dependence between flights over time.

For both the lag and auto-regressive models, the weights matrix, W, was used to account for the variable number of days between flights. In our models, $W = [\omega_{i,j}]$ where

$$\omega_{i,j} = \begin{cases} 1/(\#\text{ of days between flight } i \text{ and } j) & \text{if } 0 < i - j \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

This accounts for the 'degree of possible interaction' between flight i and j for the previous 3 flights. For these data,

$$W = \left[\begin{array}{ccccccccccccccc}
0 & 0 & & & & & & \cdots & & & & & & & 0 \\
1/14 & 0 & 0 & & & & & \cdots & & & & & & & 0 \\
1/30 & 1/16 & 0 & 0 & & & & & \cdots & & & & & & 0 \\
1/42 & 1/28 & 1/12 & 0 & 0 & & & & & \cdots & & & & & 0 \\
0 & 1/64 & 1/48 & 1/36 & 0 & 0 & & & & & \cdots & & & & 0 \\
0 & 0 & 1/61 & 1/49 & 1/13 & 0 & 0 & & & & \cdots & & & & 0 \\
0 & \cdots & & 1/62 & 1/26 & 1/13 & 0 & 0 & & & & \cdots & & & 0 \\
0 & & \cdots & & 1/40 & 1/27 & 1/14 & 0 & 0 & & & \cdots & & & 0 \\
0 & & \cdots & & & 1/41 & 1/28 & 1/14 & 0 & 0 & & \cdots & & & 0 \\
0 & & & \cdots & & & 1/98 & 1/84 & 1/70 & 0 & 0 & & \cdots & & 0 \\
0 & & & \cdots & & & & 1/98 & 1/84 & 1/14 & 0 & 0 & \cdots & & 0 \\
0 & & & & \cdots & & & & 1/99 & 1/29 & 1/15 & 0 & 0 & 0 & 0 \\
0 & & & & \cdots & & & & & 1/43 & 1/29 & 1/14 & 0 & 0 & 0 \\
0 & & & & & \cdots & & & & & 1/57 & 1/42 & 1/28 & 0 & 0 \\
0 & & & & & \cdots & & & & & & 1/56 & 1/42 & 1/14 & 0
\end{array}\right].$$

The i, j^{th} element in the matrix is equal to one over the number of days between flights i and j for the previous three flights. For example, 1/42 in the fourth row, first column indicates that there are 42 days between flights 1 and 4. If there are two predictors of net change included in the model, the model for the fourth flight (the fourth row in Equation 3.1) can be written:

$$y_4 = \beta_1 x_{14} + \beta_2 x_{24} + \rho(1/42 y_1 + 1/28 y_2 + 1/12 y_3)$$

where x_{14} is the fourth observation of the first predictor and x_{24} is the fourth observation of the second predictor. For more technical details on the lag model, see Appendix A.

Lag Model Results

To choose the best predictors of net change in sandbar size for the lag model, we considered residual plots and model fitting criteria including Akaike's Information Criterion (AIC), Akaike's Corrected Information Criterion (AICC), and Schwartz's Information Criterion (BIC) (Anselin, 1992). These criteria indicated that mean

Table 3.1: Coefficients for Lag Model. The standard deviations and p-values for the parameters associated with mean daily discharge and sand supply do not account for uncertainty in the estimate of ρ. Accounting for this uncertainty would slightly increase these p-values.

Parameter	Estimate	Std. Dev.	p-value
Intercept	-188.11	87.627	0.05
Mean Daily Discharge	0.66	0.216	0.01
Sand Supply	-29.69	39.658	0.47
ρ	-8.84	0.001	0.01
σ^2	54.91		
pseudo-R^2	0.35		

daily discharge and presence or absence of sediment supply from the LCR were the best predictors of net change. Sand supply is coded 1 if sediment was added to the Colorado River during the i^{th} interflight period, and 0 if no sediment was added.

The lag model indicates that as mean daily discharge increases and sand supply is held fixed, the average net change of sandbars increases (Table 3.1). Thus increased daily discharge is related to increasing sandbar size. The coefficient for sand supply was not significant (p-value = 0.47) but was included to facilitate comparisons with the auto-regressive model discussed in Section 3.1.2. The likelihood ratio test indicated that the lag coefficient ρ was also significant (p-value = 0.008). This indicates that the previous three flights are good predictors of the net change in sandbar size. The negative value of ρ indicates that as the weighted average of net change for the previous three flights increases, the net change in sandbar size for the current flight decreases. A relatively small amount of variability in the response was accounted for by this model, as reflected by a pseudo-R^2 of 35%. Pseudo-R^2 is the ratio of the variance of the predicted values over the variance of the observed values for the response (Anselin, 1992).

Moran's I (Anselin, 1980), a statistic used to determine whether there is significant autocorrelation between observations, indicated that there was no autocorrelation remaining in the residuals for this model (p-value=0.99). This is necessary in order for the model assumptions to hold. Standard diagnostic plots (Weisburg, 1985), such as quantile-quantile plots and plots of the residuals versus the predictors, confirmed that the assumptions of this model were reasonable. See Appendix A for more details on the lag model results.

3.1.2 Auto-Regressive Model

The auto-regressive (AR) model takes into account previous observations of the response as well as previous observations of the predictors to improve predictions about the response for the current flight. One way to interpret this model is that it takes time for the predictors to impact the size of the sandbars. In other words, this model accounts for the fact that mean daily discharge from the previous flights may also impact the response. The AR model has the following form:

$$Y = X\beta + u \text{ where } u = \rho W u + \epsilon \text{ and } \epsilon \sim N(0, \sigma^2). \tag{3.2}$$

This model can be rewritten as

$$Y = X\beta + \rho WY - \rho WX\beta + \epsilon \;\text{ with } \epsilon \sim N(0, \sigma^2). \tag{3.3}$$

W and ρ are defined as in the lag model described above (Upton and Fingleton, 1985). The auto-regressive (AR) model is sometimes called a spatial error model.

Auto-Regressive Model Results

As for the lag model, the i^{th} response for the AR model is the mean net change over the sandbars below the LCR observed on the i^{th} flight. The model selection criteria AIC, AICC and BIC indicated that mean daily discharge and sand supply from the LCR were the best predictors for the AR model. Sand supply is again coded as 1 if sediment was added to the Colorado River and 0 if no sediment was added during flight i.

Table 3.2: Coefficients for Auto-Regression Model. The standard deviations and p-values for the parameters associated with mean daily discharge and sand supply do not account for uncertainty in the estimate of ρ. Accounting for this uncertainty would slightly increase these p-values.

Parameter	Estimate	Std. Dev.	p-value
Intercept	-246.05	49.11	0.0003
Mean Daily Discharge	0.68	0.11	< 0.0001
Sand Supply	58.73	25.85	0.0423
ρ	-17.88	2.78	0.0002
σ^2	44.06		
pseudo-R^2	0.36		

The AR model again indicates that as mean daily discharge increases and sand supply is held fixed, the average net change of sandbars along the river increases (Table 3.2). Thus increased daily discharge is related to increasing sandbar size. Note that the coefficient for sand supply in the AR model is significant (p-value = 0.04). This indicates that when the LCR supplies sediment to the Colorado River and mean daily discharge is held fixed (both adjusted for the effect of the previous three flights), the net change in sandbar size increases. Thus presence of sediment is related to increasing sandbar size. The likelihood ratio test indicated that the lag coefficient, ρ, was also significant (p-value = 0.0002). This indicates that the previous three flights are good predictors of the net change in sandbar size. The negative value of ρ indicates that as the weighted average of net change for the previous three flights increases, the net change in sandbar size for the current flight decreases. As the weighted average of the predictors for the previous three flights increase, the net change in sandbar size for the current flight also increases. A small amount of variability in the response was accounted for by this AR model, as reflected by a pseudo-R^2 of 36%.

Standard diagnostic plots confirmed that the assumptions of this model were reasonable. See Appendix A for further discussion of the technical details for the AR model.

3.1.3 Limitations of Regression Models for Correlated Data

There are several limitations of the regression models for correlated data presented here. One of the primary concerns is that we cannot determine whether the parameter ρ is modeling a true dependence in the observations over time or if ρ is modeling noise due to predictors that were not measured. One of the primary causes of auto-correlated error terms is that an important predictor is not included in the model. In addition, sometimes modeling a trend over time or seasonality component will reduce or eliminate the auto-correlation in the model (Neter, *et al.*, 1990). We considered several variations of the lag and auto-regressive models and all indicated a significant lag coefficient, but with only 15 observations (one per flight) we are somewhat reluctant to make definitive conclusions regarding the lag component.

It is interesting to note that while the coefficient for sand supply is not significant in the lag model, it is *negative*. In the AR model the coefficient is significant and *positive*. The negative value for the coefficient of sand supply in the lag model at first glance is somewhat contradictory as scientific evidence indicates that as sand supply increases, sandbars should increase in size. However, exploratory plots show that sand supply may take longer than one flight period (12 day minimum) to move downriver and impact sandbars. Figures 3.3 and 3.4 show the possible lag effect of sand supply. The positive value for the coefficient of sand supply in the AR model suggests that this delayed effect of sand supply has been taken into account by weighting sand supply from the previous three flights.

Another concern with these models is that both the response and predictors are averages, and so interpretation of the models is difficult. For example, the *mean* daily discharge may not be a good measure of the water release pattern because two very different water release patterns could have the same mean daily discharge. We discuss this issue further in Chapter 5.

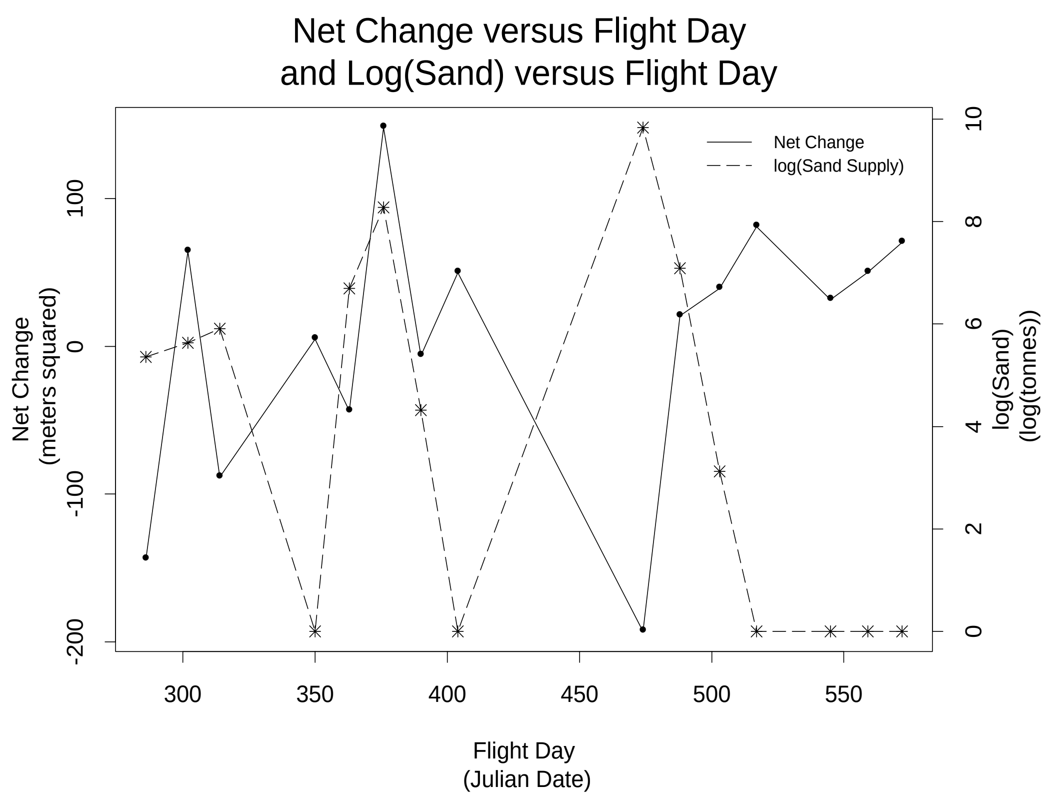

Figure 3.3: Mean Net Change (m^2) for Sandbars Below the Little Colorado River and Sand Supply (tones) Plotted Against Time (Julian days)

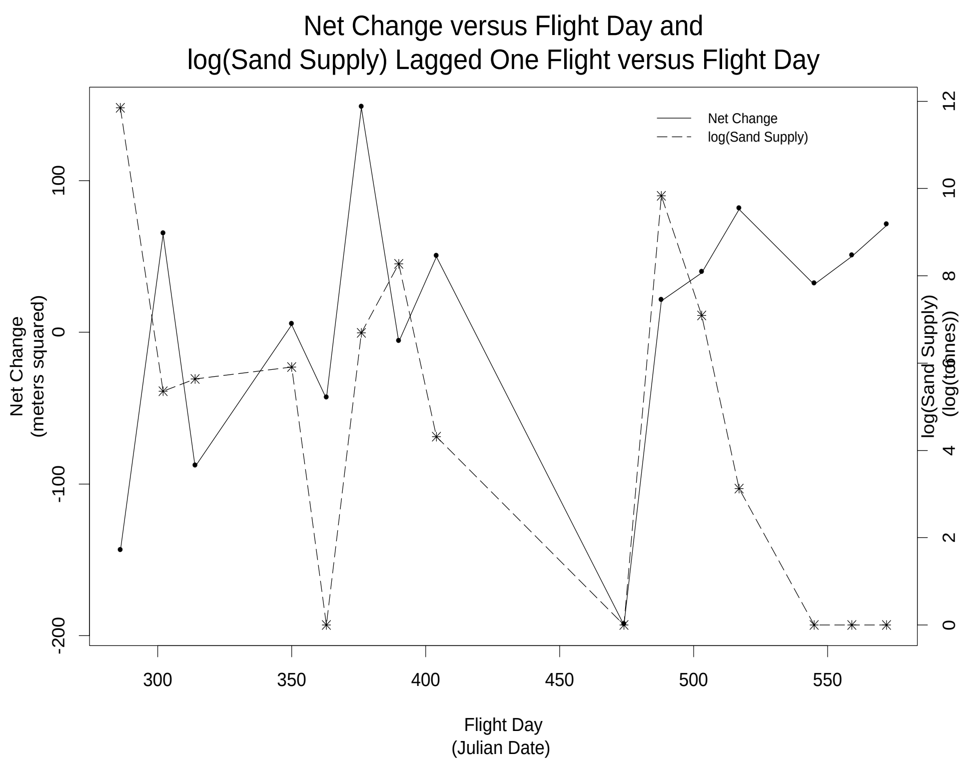

Figure 3.4: Mean Net Change (m^2) for Sandbars Below the Little Colorado River and Sand Supply (tones) Lagged One Flight Plotted Against Time (Julian days)

The main concern with these regression models for correlated data is the long and varying time intervals between observations. With netchange rangeing between -193 to 148 m^2 over sampling intervals between 12 and 70 days, we have a very incomplete picture of what is actually happening to the sandbars. In the AR model it is difficult to determine whether ρ is accounting for a trend in the data which we cannot account for because of widely varying sampling intervals, or if there truly is a temporal effect for these data. Recent data have shown that large fluctuations in sandbar area can occur in a matter of days or even several hours. Cluer (1995a) suggests that as the amount of time between observations increase, estimates of net erosion decrease. This suggests that a smaller sampling interval should be used in order to obtain a better understanding of the natural processes taking place along the river. We consider this issue further in Chapter 5.

3.2 All Sandbars

Due to limitations of the regression models for correlated data discussed above, we also considered a more straightforward linear regression model. In addition, we wanted to provide the NPS with a model which would predict the mean net change in sandbar size for all 58 sandbars sampled below the GCD, rather than just the 37 sandbars below the LCR. In the discussion that follows, the response is the average net change in sandbar size for all sandbars along the Colorado River for each flight.

The auto-correlation function for the mean net change for all sandbars in this sample (Figure 3.5) indicates there is no significant correlation over time in net change. This led to the consideration of a multiple regression model described in Section 3.2.1.

3.2.1 Regression Model

Here we describe a regression model to predict the mean net change for the entire set of sandbars along the Colorado River. The i^{th} response is the mean net

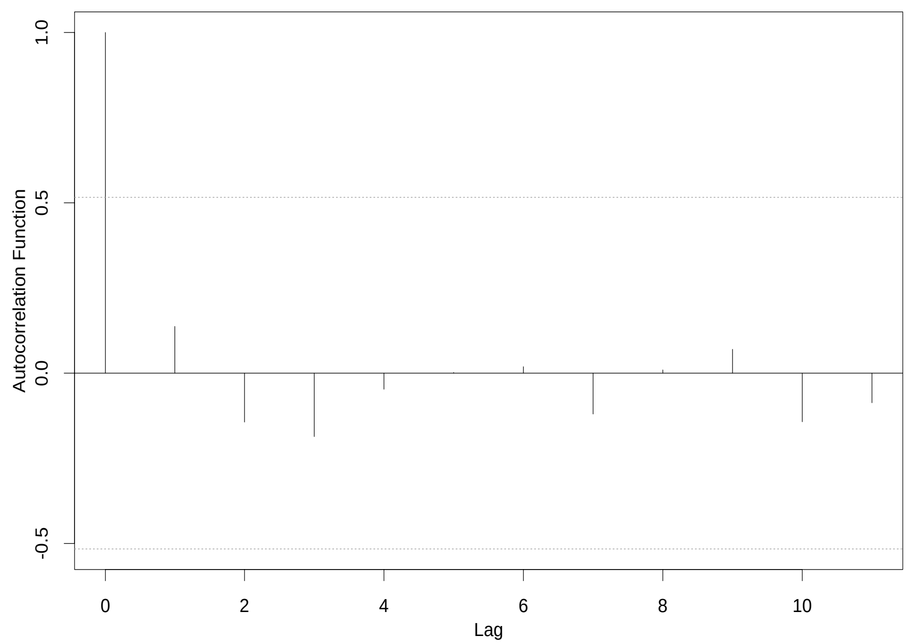

Figure 3.5: Autocorrelation Plot for Average Net Change for All Sandbars. The dotted lines indicate approximate 95% confidence interval limits. This plot implies that there is no significant lag component for these data.

change over the sandbars along the entire stretch of river observed on the i^{th} flight. The model is of the form

$$Y = X\beta + \epsilon \text{ with } \epsilon \sim N(0, \sigma^2). \tag{3.4}$$

See Appendix A for more details about the regression model.

Standard model selection procedures and residual plots suggested that mean daily discharge (Q_w) and mean daily maximum upramp (Upramp) were the best predictors of mean net change in sandbars for the entire river (Table 3.3). A relatively small amount of variability in the response was accounted for by this model, as reflected by an R^2 of 50%.

Table 3.3: Coefficients For Regression Model.

Parameter	Estimate	Std. Dev.	p-value
Intercept	-59.83	43.0	0.19
Mean Daily Discharge	0.37	0.12	0.01
Upramp	-0.38	0.17	0.05
σ	41.53		
R^2	0.50		
Model p-value	0.02		

We considered several variations of this regression model. Since mean daily discharge was consistently the best predictor of mean net change, we considered several other predictors in addition to mean daily discharge. One model which we considered included mean daily discharge and transport capacity as predictors. However, since transport capacity is determined by the amount of discharge (Equation 2.1), daily discharge would be included twice in the model making interpretation difficult. Another predictor, the range of discharge over a flight, was not a significant predictor when mean daily discharge is included in the model. Mean daily maximum downramp was a significant predictor with mean daily discharge in the model; however, upramp and downramp are highly correlated ($r = 0.74$), so we included only one of the predictors in the model. We selected mean daily maximum upramp since it resulted in slightly better residual plots. Because we averaged over each flight, per sandbar variables were not considered. Standard diagnostic plots confirmed that the assumptions of this model were reasonable. See Appendix A for further technical details.

3.2.2 Limitations of Regression Model

The regression model suggests that as mean daily discharge increases and upramp remains fixed, mean net change also increases. So, increased discharge is related to increased sandbar size. However, as upramp increases and mean daily discharge remains fixed, average net change decreases (i.e., sandbars decrease in size on average). You may recall that upramp is the mean of daily maximum upramp at the five gauging stations averaged over the flight period. The coefficient for upramp is difficult to interpret because the amount of upramp increases with distance from the dam. In other words, the gauging station farthest downriver will show a more rapid increase in discharge level than one close to the dam for any given increase in flow. Because upramp is the average of five gauging stations along the river, there is no way to account for any higher upramp observed farther downriver.

Another limitation of the model is that the response is *mean* net change for all sandbars on each flight. When the response is an average, correlations between the response and the predictors can be inflated. This is called ecological correlation (Freedman, *et al.*, 1980). Averaging reduces data variability which can lead to the impression of a closer fit for a regression model.

Chapter 4

CONCLUSIONS

The primary goal of this study was to provide the resource managers of interested agencies with a model to predict the change in area of sandbars downriver from the GCD. We found that the mean daily discharge from the GCD and mean daily maximum upramp were good predictors of net change. The best predictors for the regression models for correlated data for net change in sandbars below the LCR were mean daily discharge and the presence or absence of sand supply from LCR. In the regression models for correlated data there was also a lag term for the response and the predictors. The average net change was influenced by the previous three observations, indicating that antecedent conditions of flow and sand supply are important predictors, but their effects are relatively brief.

Since the regression models for correlated data for sandbars below the LCR include mean daily discharge for the GCD and presence or absence of sand supply from the LCR as predictors, these models might provide a better understanding of the underlying natural processes of the river as compared to the more straightforward regression model. However, with so few observations, we are wary of making any definitive conclusions from the regression models for correlated data. The models suggest that more systematic and larger samples could reliably predict change in sandbar size with these predictors.

Since the regression model for all sandbars along the Colorado River is a more straightforward model, we do not need to be quite as concerned about the small sample size. This model indicates that as mean daily discharge increases and upramp remains fixed, the average net change of sandbars along the river increases. In other words, the sandbars tend to increase in size on average when daily discharge increases. As mean daily maximum upramp increases and mean daily discharge remains fixed, the average net change of sandbars along the river decreases. In other words, the sandbars tend to decrease in size on average when daily maximum upramp increases.

Chapter 5

MANAGEMENT CONSIDERATIONS

This study provides information useful to management as research and monitoring in the Grand Canyon enters a phase of long-term monitoring. For future studies it may be useful to use alternate sampling plans to improve accuracy of the resulting estimates and to possibly reduce study costs. Several suggestions for future studies are provided below.

5.1 Systematic Stratified Sampling

A fundamental goal of survey design is to maximize the amount of information collected for a fixed cost. The data described in this report were collected using a design that could be considered to be a simple random sample of 58 sandbars out of the population of approximately 600 sandbars.

For more complex space/time problems, such as determining the impact of dam water releases on sandbars in the Colorado River, it can be useful to consider more complex sampling plans. A stratified sample is one obtained by separating the population into nonoverlapping groups, called strata, and then selecting a sample from each stratum (Scheaffer *et al.* 1979). Stratified sampling has the advantages that the data within each stratum should be more homogeneous than the entire population of data, estimates of population parameters within each stratum can be obtained without additional sampling, and the cost of conducting the sampling may be reduced (Scheaffer *et al.* 1979). Stratified sampling has been used to collect data in other spatial problems. For example, the National Resources Inventory (NRI), a survey to "assess soil characteristics, land use, erosion, and conservation needs for all nonfederal land in the U.S., Puerto Rico, and the U.S. Virgin Islands", uses a stratified sampling design (Cox *et al.*, 1995).

Konijn (1973) suggests systematic stratified sampling as a method for collecting spatial data. Systematic sampling entails sampling at regular repeated intervals. For studies on sandbar size, systematic stratified sampling would entail sampling systematically over time, while stratifying the river in some manner. The method of stratification also needs to be determined. Green (1979) suggests that for large scale environmental patterns, the area should be broken up into relatively homogeneous subareas. One possible method is to stratify the river into homogeneous areas by river mile, by sandbar size, or perhaps by geomorphic characteristics such as proximity to sediment supplying tributaries, with depth ratio, gradient or geologic controls.

In order to determine the sampling interval for systematic stratified sampling, the number of observations that can be collected in a year needs to be determined. Since economic considerations are usually an overriding factor, the cost of collecting each observation will need to be considered. The costs will depend upon the method of data collection and could possibly involve or combine several methods of data collection such as helicopter flight, land surveying, or automatic cameras.

Once the sampling interval and stratification are determined, the number of observations from each strata needs to be determined. One way to determine the number of observations per strata is to use proportional allocation. This method allows for different size strata, by keeping the sample fractions, $f_h = n_h/N_h$ equal for all strata. N_h is the population size of strata h, and n_h is the sample size for strata h. (Cochran, 1977)

5.2 Other Suggestions for Future Studies

Ironically, the time intervals between observations in this study were much shorter than typically used for monitoring sandbars in the Grand Canyon and yet one of the primary limitations of this study was the long time intervals between observations. This made it difficult to account for a considerable amount of the variability in net change. Figure 5.1, a hypothetical example similar to Figure 7 in Cluer (1995a), shows that large sampling intervals can lead to erroneous conclusions. The plot shows that a great deal of information can be lost when sampling over a longer interval. Daily observations on this hypothetical example would show an increase in sandbar area, then a sudden drop in area. Observations over a longer period simply show a negative trend in sandbar area and would underestimate the variation evident in the daily measurements. Smaller sampling intervals would allow for a better understanding of the natural processes and responses to dam operations.

More frequent observations over fewer sandbars could also save money. Aerial photography is one method of data collection for studies of this type. However, flying a helicopter through the Grand Canyon at a low altitude is costly, ecologically unsound, and possibly dangerous. Another traditional method of data collection is land surveying which is similarly expensive and very time consuming. An alternate sampling method would be to set up automatic cameras along the river which would take pictures of a few sandbars at specified intervals. While limiting the number of sandbars in the study, daily observations from fixed cameras would increase the amount of information gained about the problem of interest. This method has been used on a small number of sandbars since 1993 (Cluer and Dexter 1994, Dexter and Cluer 1995, Dexter *et al.* 1995) with promising results.

With any study of sandbar size there is a trade-off between collecting larger samples (i.e., more sandbars included in the study) and collecting more frequent measurements of the sandbars included in the study. It is important to include

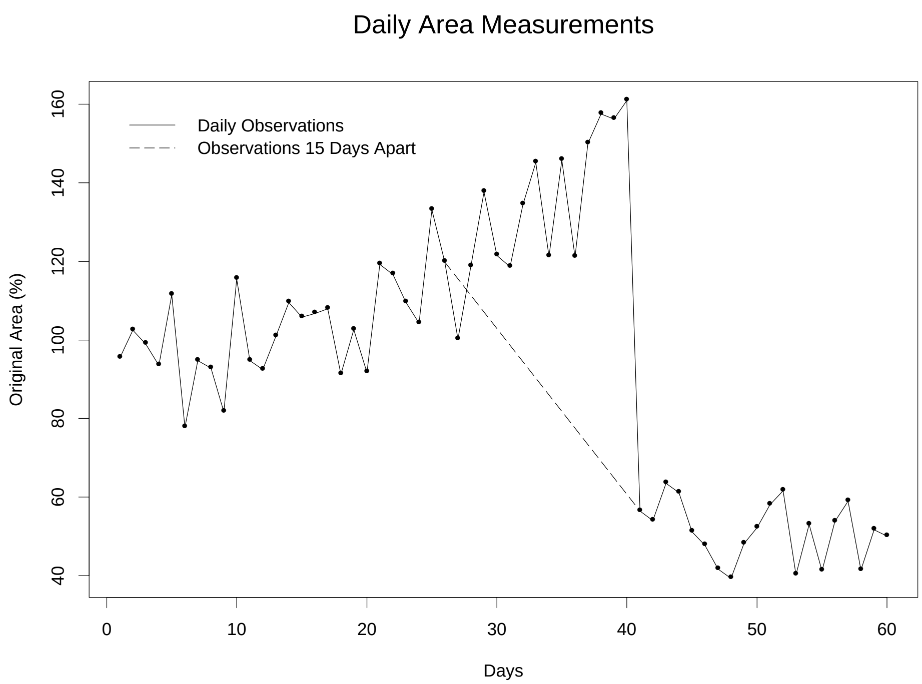

Figure 5.1: Plot of Hypothetical Sampling Example. This is a hypothetical data set to display the amount of information that can be missed when observations are recorded every 15 days as opposed to daily records.

enough sandbars in the study so the sample is representative of the population of sandbars along the Colorado River and so statistical estimation is possible. However, collecting more frequent observations should be the primary consideration for future studies. This will greatly increase scientists' understanding of how changes in sandbar size are related to the natural processes of the river and to dam operations. Future studies should be designed with the dual goals of having enough sandbars to constitute a representative sample and to allow for statistical modeling, while collecting the observations at small enough time intervals to allow for a better understanding of the relationship between changes in sandbar size and hydrologic, geomorphic, and geographic parameters.

Two additional considerations for future studies are related to potential statistical models and interpretations of these models. First, one of the limitations of the models presented here is the fact that we used *means* as both predictors and the response (averages of all observations collected for each flight). In future studies we suggest further consideration of exactly what these means represent. For example, mean daily discharge is the average discharge from the dam over a flight period. However, two very different flow patterns could have the same mean. A constant flow could have the same mean as a widely fluctuating flow. In these situations, the mean alone does not represent the flow pattern. In addition, the response, mean net change in sandbar size, is only representative of the *average* net change in sandbar size. Mean net change does not reflect the fact that some sandbars change drastically over a flight period, while others only change slightly. The daily sampling plan suggested above would largely resolve this problem.

A second issue we did not consider here is whether or not the predictors are measured with error. We assumed that the predictors were fixed (measured without error). If it is more reasonable to assume that these predictors were measured with error, this uncertainty should be incorporated in future models.

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We would like to acknowledge the help of Bill Vernieu (GCES) who provided the flow data and Greg Fisk (USGS) who provided the Paria River and LCR sediment flow data.

References

[1] Anselin, L. (1980), *Estimation Methods for Spatial Autoregressive Structures*, Regional Science Dissertation and Monograph Series, Ithaca, New York.

[2] Anselin, L. (1992) *Spacestat Tutorial: A Workbook for Using* SpaceStat *in the Analysis of Spatial Data*, Department of Geography Technical Software Series S-92-1.

[3] Brown, B.T.; Johnson,R.R. (1987), "Fluctuating Flows from Glen Canyon Dam and Their Effect on Breeding Birds of the Colorado River," National Park Service Technical Report Number GCES/23/87.

[4] Cluer, B. (1995a), "Cyclic Fluvial Processes and Bias in Environmental Monitoring, Colorado River in Grand Canyon," *The Journal of Geology*, **103**, 411-421.

[5] Cluer, B. (1995b), "Aerial Photography of the GCES-II Test Flows and Interim Flows," Final Report, U.S. Department of Interior, National Park Service. Flagstaff, AZ.

[6] Cluer, B.L. and Dexter, L.R. (1994), "Daily dynamics of Grand Canyon sandbars; monitoring with terrestrial photogrammetry." Final Report, U.S. Department of Interior, National Park Service. Flagstaff, AZ. 200p.

[7] Cochran, William G. (1977), *Sampling Techniques,* Wiley and Sons, New York.

[8] Cox, D.D.; Cox, L.H.; and Ensor, K.B. (1995), "Spatial Sampling and the Environment," National Institute of Statistical Sciences Technical Report no. 38.

[9] Dexter, L.R. and Cluer, B.L. (1995), "Extension study: daily dynamics of Grand Canyon sandbars; monitoring with terrestrial photogrammetry." Draft Final Report, U.S. Department of Interior, National Park Service. Flagstaff, AZ. 121p.

[10] Dexter, L.R., Cluer, B.L., and Manone, M.F. (1995), "Using land based photogrammetry to monitor sandbar stability in Grand Canyon at a daily time scale." In, Van Riper, C., III, ed.: *Proceedings of the Second Bienial Conference of Research in Colorado Plateau national Parks.* National Park Service, p. 67-86.

[11] Freedman, D; Pisani, R; Purves, R (1980), *Statistics*, W.W. Norton and Co., New York.

[12] Green, Roger H. (1979), "Sampling Design and Statistical Methods for Environmental Biologists," John Wiley and Sons, New York.

[13] Kearsley, L.H.; Schmidt J.C.; and Warren K.D. (1994), "Effects of Glen Canyon Dam on Colorado River Sand Deposits Used as Campsites in Grand Canyon National Park, USA," *Regulated Rivers: Research and Management*, **9**, 137-149.

[14] Konijn, H.S. (1973), *Statistical Theory of Sample Survey Design and Analysis,* American Elsevier Publishing Co., New York.

[15] Maddux, H.R.; Kubly, D.M.; deVos, J.C. Jr.; Persons, W.R.; Staedicke, R.H.; Wright, R.L. (1987), "Evaluation for Varied Flow Regimes on Aquatic Resources of Glen and Grand Canyons," U.S. Department of Interior Technical Report GCES/12/87.

[16] Neter, J.; Wasserman, W.; Kutner, M. (1990), *Applied Linear Statistical Models*, Richard D. Irwin, Inc., Illinois.

[17] Ord, K. (1975), "Estimation Methods for Models of Spatial Interaction," *Journal of the American Statistical Association*, **70:349**, 120-126.

[18] Scheaffer, R.L.; Mendenhall, W., Ott, L. (1979) Elementary Survey Sampling, Duxbury Press: North Scituate, MA.

[19] Smillie, G.M., Jackson, W.L., and Tucker, D. (1992), "Colorado River Sand Budget: Lees Ferry to Little Colorado River," National Park Service Technical Report NRWRD/NRTR-92/12, 11pp.

[20] Upton, G.; Fingleton, B. (1985), *Spatial Data Analysis by Example; Vol. 1: Point Pattern and Quantitative Data*, John Wiley and Sons, New York.

[21] Weisburg, Sanford (1985), *Applied Linear Regression*, John Wiley and Sons, New York.

Appendix A

TECHNICAL APPENDIX

A.1 Lag Model

The lag model described in Section 3.1.2 has the form:

$$Y = X\beta + \rho WY + \epsilon \text{ with } \epsilon \sim N(0, \sigma^2). \tag{A.1}$$

The maximum likelihood estimates for the parameters in this model are given below.

$$\begin{aligned} \hat{\beta} &= (X^T X)^{-1} X^T (I - \hat{\rho} W) Y, \\ \hat{\sigma}^2 &= \frac{1}{n} \left[Y^T (I - \hat{\rho} W)^T (I - X(X^T X)^{-1} X^T)(I - \hat{\rho} W) Y \right], \end{aligned}$$

and $\hat{\rho}$ is the value of ρ that minimizes

$$-2n^{-1} ln|(I - \rho W)| + ln\hat{\sigma}^2.$$

If ρ is known, the covariance of $\hat{\beta}$ is $cov(\hat{\beta}) = \sigma^2 (X^T X)^{-1}$. This covariance does not account for uncertainty in the estimate of ρ. Full details of the derivations can be found in Upton and Fingleton (1985).

For both the lag model (4.1) and the auto-regressive model described below, we used the computational procedure discussed by Ord (1975) to estimate ρ. To compute the determinant of the correlation matrix, $|I - \rho W|$, $|I - \rho W|$ can be written $\sum_{i=1}^{n} (1 - \rho \omega_i)$ where ω_i are the eigenvalues of the weights matrix, W. To compute

$\hat{\rho}$, we used the nonlinear maximization function *nlmin*[1] in the statistical computing package *S-plus*©. This allows for a straightforward estimation of ρ.

The weights matrix, W, described in Section 3.1.1, is typically "normalized" so that the rows of the matrix sum to 1. This gives ρ, a measure of correlation, a natural interpretation as $|\rho| < 1$. We chose not to normalize the weights matrix because the relationship between different columns would be lost. For example, normalizing W would give observations that were 14 and 28 days apart the same relationship as observations that were 32 and 64 days apart. Because we did not normalize the weights matrix, the estimate of ρ does not have a natural interpretation.

For the lag model described in Section 3.1.1 standard diagnostic plots (Weisburg, 1985), such as quantile-quantile plots and plots of the residuals versus the predictors, confirmed that the assumptions of this model were reasonable (Figure A.1). There were a few issues to consider when interpreting the residual plots. The plot of the residuals versus mean daily discharge shows a possible pattern of decreasing variability in the residuals as mean daily discharge increases. We considered several transformations both on mean daily discharge and average net change, and this residual plot never improved greatly. One possible explanation for this is the fact that there are only two observations with mean daily discharge greater than 450 m^3/s. Thus, the decreased variability may just be an artifact of the distribution of the data. The plot of the residuals versus sand supply shows a possible pattern remaining in the residuals, however the difference between sand levels does not appear to be statistically significant.

In Section 3.1.1 we describe Moran's I (Anselin, 1980), a statistic used to determine whether there is significant autocorrelation between observations. Moran's I

[1] The coefficients for these models were computed using functions from the spatial library created by Richard A. Davis and Robin Raich at Colorado State University.

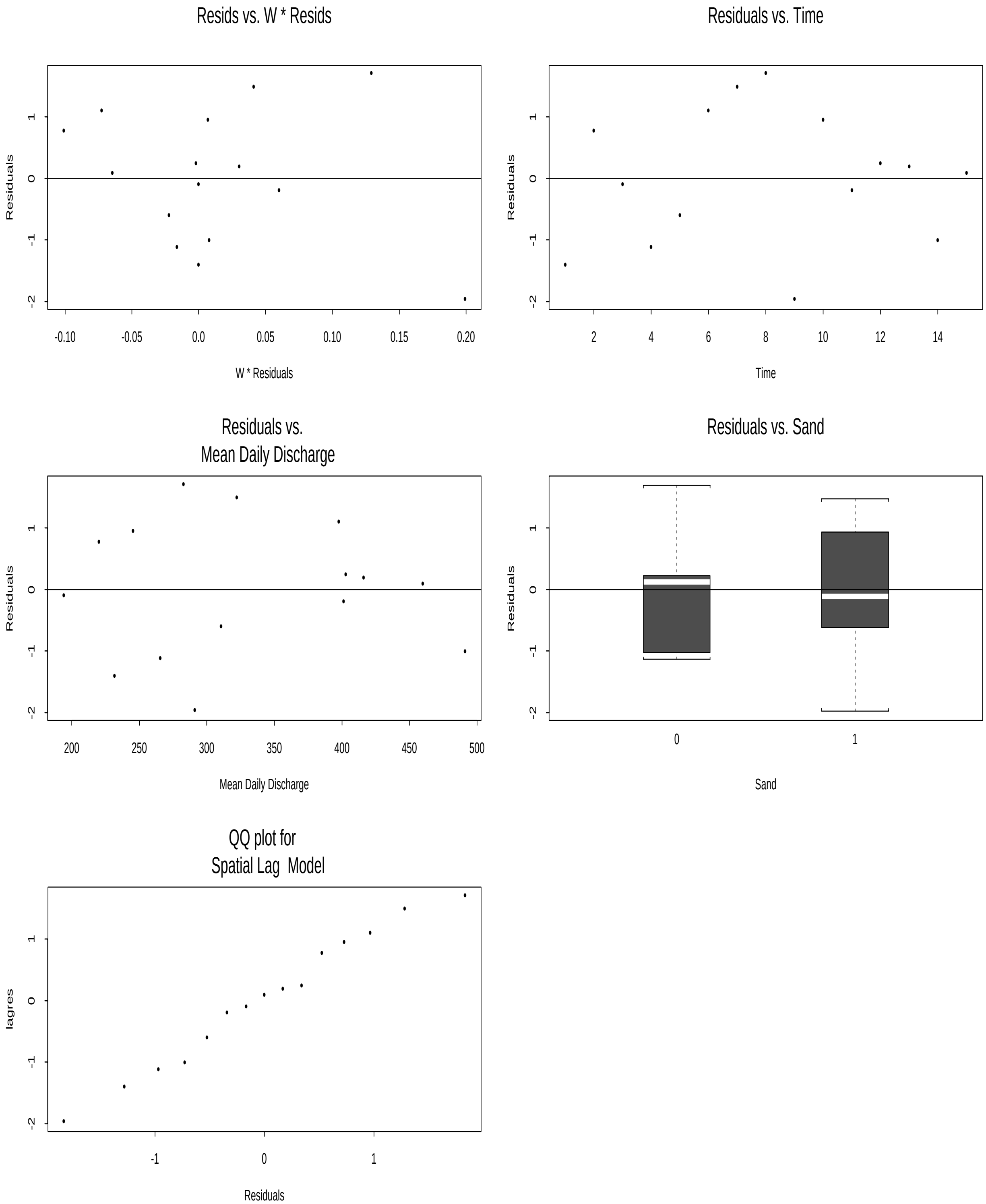

Figure A.1: Residual Plots for the Lag Model.

indicated that there was no significant autocorrelation remaining in the residuals (p-value=0.99). We used the randomization approach to compute the p-value for Moran's I because of the small sample size (15 observations). This lack of correlation is supported by the plot of the residuals versus the weights matrix multiplied the residuals (Figure A.1).

A.2 Auto-Regressive Model

The auto-regressive model described in Section 3.1.2 has the form:

$$Y = X\beta + u \text{ where } u = \rho W u + \epsilon \text{ and } \epsilon \sim N(0, \sigma^2). \tag{A.2}$$

The maximum likelihood estimates for the parameters in this model are given below. Full details of the derivations can be found in Ord (1975) and Anselin (1980).

$$\begin{aligned} \hat{\beta} &= [X^T(I - \hat{\rho}W)^T(I - \hat{\rho}W)X]^{-1}[X^T(I - \hat{\rho}W)^T(I - \hat{\rho}W)Y, \\ \hat{\sigma}^2 &= \frac{1}{n}\left[[Y - A(I - \hat{\rho}W)Y]^T(I - \hat{\rho}W)^T(I - \hat{\rho}W)[Y - A(I - \hat{\rho}W)Y]\right], \end{aligned}$$

where $A = X(X^T(I - \hat{\rho}W)^T(I - \hat{\rho}W)X]^{-1}X^T(I - \hat{\rho}W)^T$, and $\hat{\rho}$ is the value of ρ that minimizes

$$-2n^{-1}ln|(I - \rho W)| + ln\hat{\sigma}^2.$$

If ρ is known, the covariance of $\hat{\beta}$ is $cov(\hat{\beta}) = \sigma^2[X^T(I - \rho W)^T(I - \rho W)X]^{-1}$. This covariance does not account for uncertainty in the estimate of ρ.

For the auto-regressive model described in Section 3.1.2, standard diagnostic plots confirmed that the assumptions of the model were reasonable (Figure A.2). Again the plot of the residuals versus mean daily discharge shows a possible pattern of decreasing variability in the residuals as mean daily discharge increases. We considered several transformations both on mean daily discharge and average net change, and the residual plot never improved greatly. One possible explanation for this is the fact that there are only two observations with mean daily discharge greater than 450 m^3/s. The plot of the residuals versus sand supply shows a possible remaining trend; however, once again it does not appear to be statistically significant. Moran's I statistic under the randomization approach again indicated that there was no significant autocorrelation remaining in the residuals (p-value= 0.71). This result is again supported by the residual plot of the residuals versus the weights matrix multiplied by the residuals.

We also considered other weights matrices that would supply a different lag component. We examined lags of one and two flights. The residual plots and model fitting criteria suggested that the lag three model was the best model.

Since the response is *average* net change with unequal numbers of sandbars per flight, we also considered a weighted form of both the lag and AR models. In the weighted model the variances of the errors were assumed to be different. Thus, ϵ was distributed $N(0, \sigma^2\Sigma)$, where $\text{var}(\epsilon_i) = \frac{\sigma^2}{n_i}$ and n_i is the number of observations included in the average for the i^{th} response. After examining the estimates and residual plots for both of these weighted models, we determined that weighting did not significantly improve the models. Therefore, we chose to focus on the more straightforward unweighted models.

A.3 Regression Model

The linear regression model discussed in Section 3.2.1 has the form:

$$Y = X\beta + \epsilon \ \text{ with } \epsilon \sim N(0, \sigma^2). \tag{A.3}$$

The least squares estimates for the parameters in this model are given below. Full details of the derivations can be found in Weisburg (1985).

$$\begin{aligned} \hat{\beta} &= [X^T X]^{-1} X^T Y, \\ \hat{\sigma}^2 &= \frac{1}{n-p}\left[Y^T(I - X(X^T X)^{-1} X^T)Y\right]. \end{aligned}$$

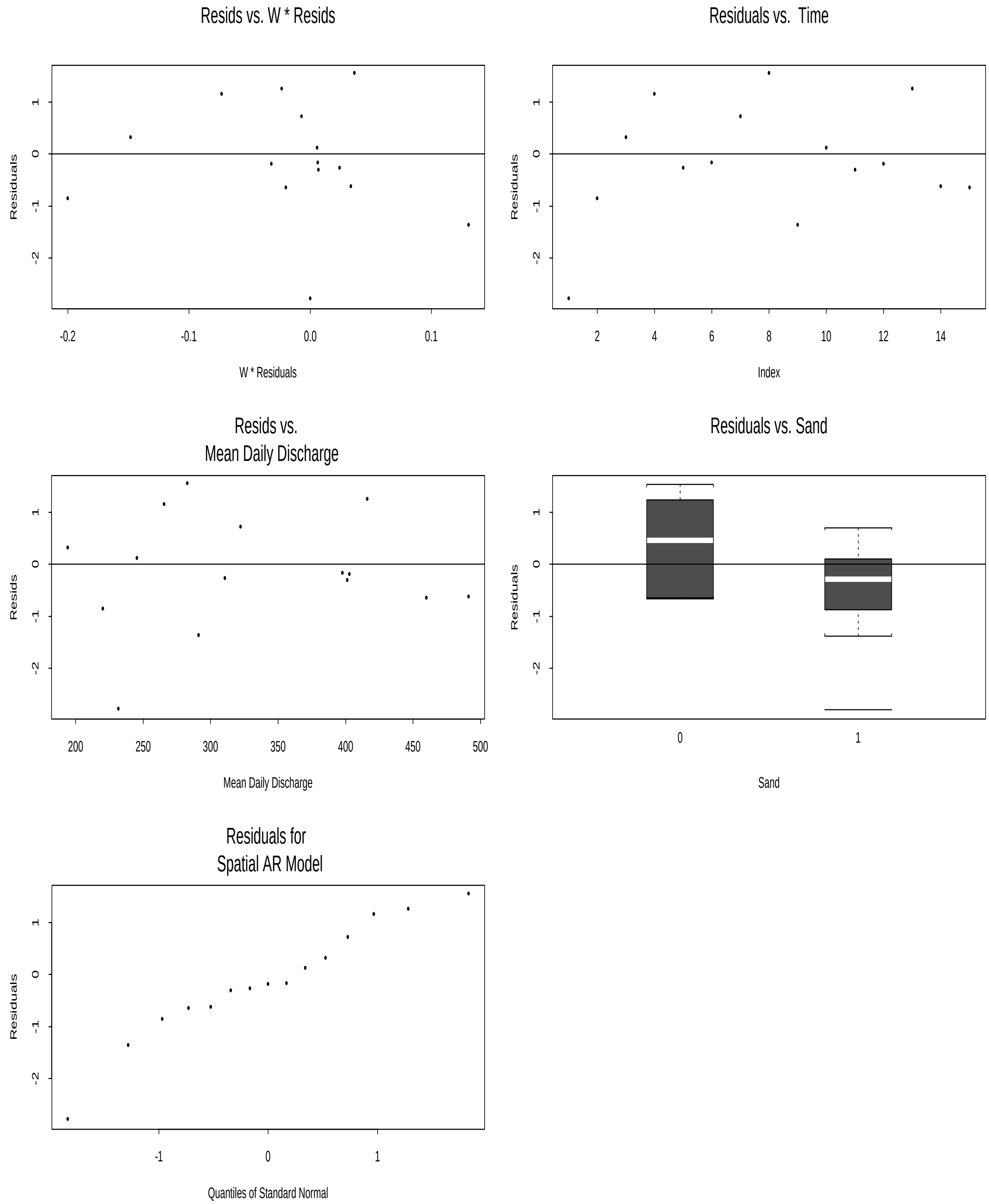

Figure A.2: Residual Plots for the Auto-Regressive model.

In Section 3.2.1 we discuss a model with mean daily discharge (Q_w) and mean daily maximum upramp (Upramp) as predictors of mean net change for all sandbars in the study. Standard diagnostic plots confirmed that the assumptions of this model were reasonable (Figure A.3). Cooks Distance, a diagnostic which indicates influential observations, indicated that the observations denoted by an empty circle in Figure A.3 were influential observations. This means that these observations have a large impact on estimates of β. Removing these two observations resulted in a better fitting model. Removing the influential observations from the regression model reduces the standard errors of the coefficients and gives more significant coefficients (Table A.1). R^2 also increased from 50% to 68% when these observations were removed. The estimate of σ was reduced to 29.27, a considerable reduction from the model which included the influential observations, when $\sigma = 41.53$. The residual plots for this model, shown in Figure A.4 show a slight improvement in the quantile quantile plot as compared to the regression model with all observations. Note that no residuals have an absolute value greater than 2.5 indicating that there are no outliers in the data. Since the estimates for the coefficients of the model with the influential observations removed were quite close to the estimates for the coefficients of the model with the influential observations included, we did not consider the model results reported in Table A.1 further.

Table A.1: Coefficients For Regression Model With Influential Observations Removed

Parameter	Estimate	Std. Dev.	p-value
Intercept	-88.61	33.20	0.024
Mean Daily Discharge	0.44	0.10	0.001
Upramp	-0.33	0.13	0.031
σ	29.27		
R^2	0.68		
Model p-value	0.003		

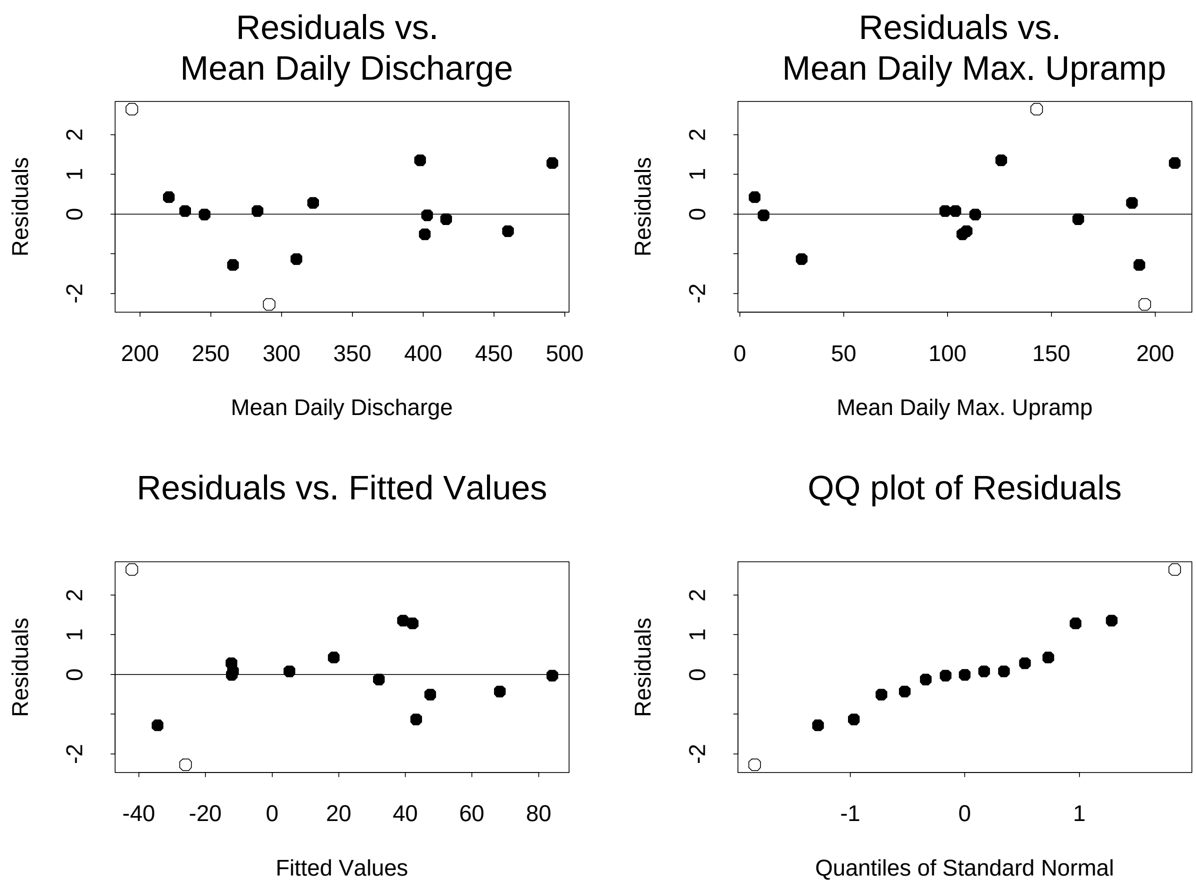

Figure A.3: Residual Plots for Model with All Observations. The hollow points are influential observations.

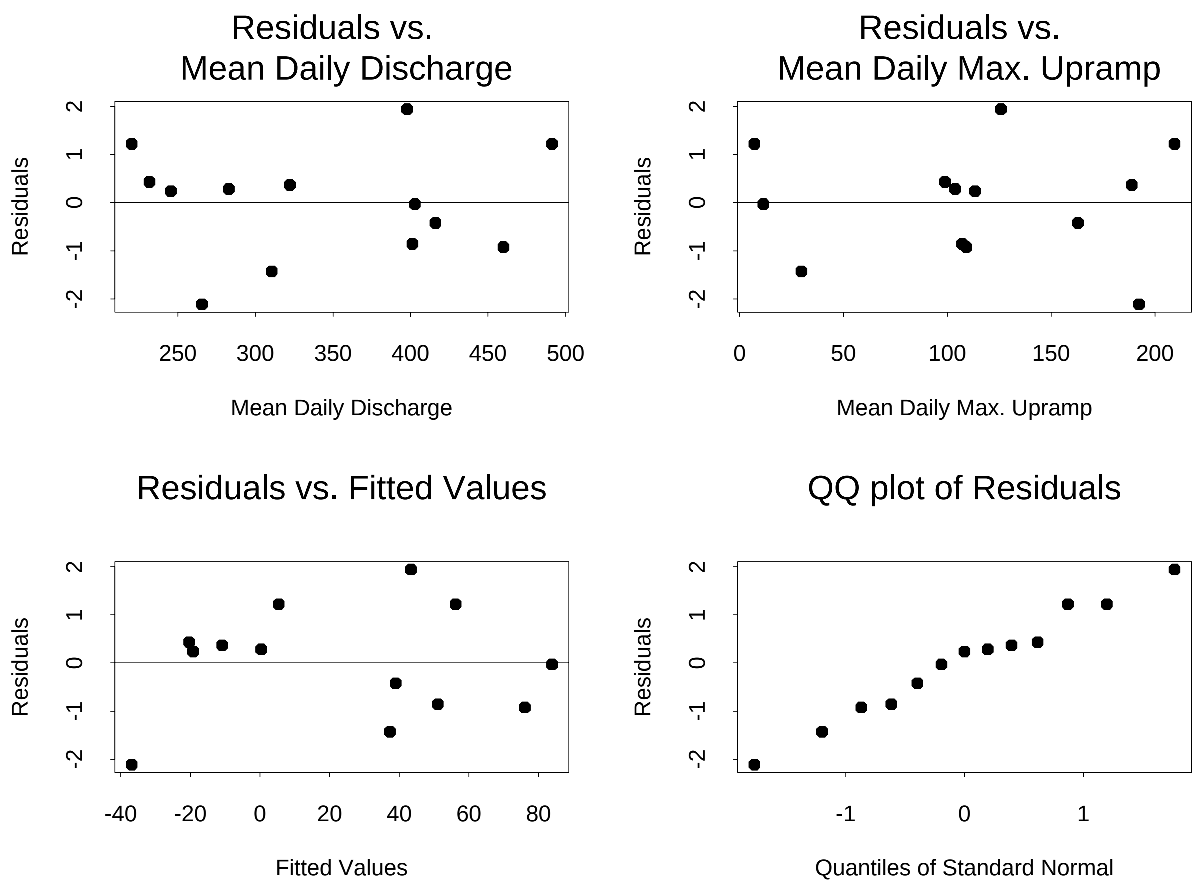

Figure A.4: Residual Plots for Model with Influential Observations Removed

Since the response is *average* net change, with unequal numbers of sandbars per flight, we also considered a weighted regression model, where $\text{var}(\epsilon_i) = \frac{\sigma^2}{n_i}$ as described above. After examining the estimates and residual plots for this model, we determined that weighting did not result in a large improvement in the model. Therefore, we chose to focus on the unweighted regression model.

Appendix B

BUDGET

The project was funded for $10,650.

Description	Cummulative Amount
Personnel	$9,211.21
Domestic Travel	$0.00
International Travel	$0.00
Materials and Supplies	$53.25
Subcontracts	$0.00
Equipment	$0.00
Indirect Cost @ 15%	$1,385.54
Total	$10,650.00

Appendix C

RESEARCH DATA