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A Conversation with Murad S. Taqqu (In Memoriam)

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Abstract. Murad S. Taqqu was Professor Emeritus of Mathematics and Statistics at Boston University, where he has spent nearly 40 years. He received his Ph.D. in Statistics from Columbia University in 1972, and after spending two years at the Hebrew University in Jerusalem and the Weizmann Institute, Israel, he joined Cornell, where he rose to the rank of a full professor. Murad's work spans a wide range of theoretical and applied topics, including long-range dependence, heavy-tailed phenomena, self-similarity, and noncentral limit theorems. His research saw no boundaries between the fields of probability, statistics, or their applications. It was motivated by and applied to real-world problems in hydrology, finance, and computer communications, where he has made profound contributions. Over his career, Murad has published four monographs, seven edited volumes and more than 270 papers, which have shaped and often created important research directions in probability and statistics. He has supervised 15 Ph.D. students and has actively collaborated with many junior as well as prominent senior researchers.

This paper is based on a series of interviews with Murad in October and November of 2023. It begins with some personal details about his complex life as an emigrant, which took him from Iraq to Lebanon, Italy, Switzerland, and eventually the United States. It discusses his graduate studies at Columbia University, and his work with and influence by Benoit Mandelbrot. It then goes over just a few landmarks of his prolific research career. The discussion includes stories about Murad's academic life as a student and faculty at EPFL, Cornell, Boston and other places. Some of the characters in these stories include Ernst Stueckelberg, Georges de Rham, Burton Singer, Benoit Mandelbrot, Murray Rosenblatt, Eugene Dynkin, Kiyoshi Itô, Samuel Karlin, Henry McKean, as well as, Peter Major and Roland Dobrushin, alongside Murad's students and collaborators. The authors of this interview regret they have not been able to mention all of Murad's many friends and collaborators. Sadly, Murad S. Taqqu passed away on April 28, 2026, at the age of 84.

Key words and phrases: Long-range dependence, self-similarity, heavy tails, fractional Brownian motion, noncentral limit theorems, stable processes.

1. EARLY YEARS AND EDUCATION

1.1 From Iraq to Switzerland

Stilian Stoev: You were born in 1942, during World War II, to a Jewish family living in Baghdad, Iraq. You then

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FIG. 1. *Murad and Rachelle Taqqu* (*photo by Norma Terrin*).

moved to Switzerland at an early age before coming to the United States for your graduate studies. Had your family been living in Iraq for long? What made them emigrate to Europe and when?

Murad Taqqu: They had been in Iraq for a long time, probably two millennia like the rest of the Iraqi Jewish community. My mother, at least, has a known genealogy of about six or seven generations. One of her ancestors was a very famous rabbi, Hakham Yosef Hayim, and because he is very famous, there are lots of things known about him. And so you can trace her lineage a long way back and I can say that they lived in Iraq for many, many generations, but I don't know exactly how many.

Stilian Stoev: How big was the Jewish community in Iraq when you were there?

Murad Taqqu: It was quite large. I'm not sure at this point. But what happened really is that before the war, the Second World War, there was a pogrom against the Iraqi Jewish community where more than six hundred people died overnight. My wife, Rachelle (Figure 1), who has a Ph.D. in history, can explain that better.

Rachelle Taqqu: I will put on my historian's hat and just tell it to you. Murad is referring to something that happened before he was born, around the time that his parents got married. During the Second World War, when the British occupied Iraq, there were Iraqi nationalists who were Nazi sympathizers, who fomented riots and targeted Iraqi Jews. They managed murdering more than six hundred people in twenty four hours. This tragic event is called the *farhod* or *farhud*—a matter of transliteration (see, e.g., [48]).

And so, his father told me that they got married around the same time. He was a very shrewd man, his father. He was a businessman, and he said to me in Arabic, which I barely understood, that at that point he knew that they were not staying in Iraq.

Murad Taqqu: Thank you, Rachelle. So, what happened is that around the time I was four years old, we left for Lebanon. This was a route that a lot of Jews took. Each one followed the other, and so the thing to do was to leave Iraq and go somewhere else. The natural place to go to was Lebanon, and so we spent two years in Lebanon.

There, I went to school, which was a French school in Lebanon. But I remember that there were also riots on the streets in Lebanon against the Jews. One of my recollections was that there, we were in a house which was basically going to be put on fire, and somebody came and picked me up to get me out of the house. So that's a recollection from when I was around 5 or 6 years old....

And then in forty eight (1948), when the state of Israel was created, there was a lot of retaliation against the Jews in Lebanon in particular. So we ended up leaving Lebanon and going to Italy. We took more or less the last ship for Italy. They had almost closed the Mediterranean at the time, so you couldn't go anywhere. You couldn't travel, but the last available ship was to Italy. So we took it. That was again a natural route for Jews who were leaving Lebanon and so we ended up in Italy.

Italy! Basically, it was okay to live there, to stay there, but we had a tourist visa, which meant that after a certain period of time you had to leave. One way or the other, we ended up staying in Milano for two more years, and then we couldn't stay there anymore.

So this is how we ended up in Switzerland, and then I went to school in Switzerland. I attended French schools, because as you know, Switzerland has French, German, and Italian regions. We were in the French part and I ended up in the school system there, eventually going to the gymnasium school system in Switzerland.

Stilian Stoev: So you got to Switzerland by the age of eight and went to primary and high schools there, correct?

Murad Taqqu: Yes, I was eight, and it was complicated. We ended up going to private boarding schools at first. We hardly had any money. So my father chose schools that were in financial trouble because they were not expensive. We ended up attending a series of failing schools where we didn't attend many classes.

Stilian Stoev: Can you tell us a little bit more about the high school system in Switzerland? How come an immigrant with irregular education like you ended up going to one of the top universities in the world—*École Polytechnique Féderale de Lausanne*?

Murad Taqqu: Because I went to dubious private schools, I was outside of the school system and I really was not learning much. In order to go to the gymnasium, which is a kind of high school essentially, you had to pass an exam. I applied to go to high school (gymnasium), because without going to high school, you can't go to university. There was only one gymnasium for the whole canton. Switzerland is divided into parts (cantons), and the education depends on which part you are in. We happened to be in a canton where everyone spoke French, so the gymnasium was in French. I worked very hard, so that really after waiting a year, I ended up being able to go to the gymnasium. For instance, one of the languages that you had to learn for the gymnasium was German. We lived in the French part of Switzerland, but the other most important area is the German one. However, the students in the French area did not like German because they were all French-educated and so they hardly could speak it. Because I liked languages, I ended up learning German on my own at a satisfactory level rather quickly, and I was able to enter the high school system this way. At the end, there is an exam like Baccalauréat in France, Maturité Fédérale, in the French part of Switzerland. I ended up doing very well, so that allowed me to get into the higher education system.

I liked school very much because I'd never entered a regular school. The gymnasium was a very advanced place. It was the only one, I think, in the whole canton de Vaud. And there you had to choose whether you wanted to study Greek and Latin, or Science. I was obviously not able to do Latin because I didn't know it at all. It was clear that I was going to study math, physics and science. The teachers were typically university lecturers who couldn't find a job at the university. So they went to this gymnasium to teach. They were very good. When there was an open position at the university they would move from the gymnasium to the university.

1.2 Languages

Vladas Pipiras: Can I ask you about the languages you speak?

Murad Taqqu: I got Italian from Italy.

Vladas Pipiras: But that was just two years, wasn't it?

Murad Taqqu: Yes, that was two years essentially and the school I went to in Italy was also a French school, and so I had studied French in Italy. However, in Italy everyone speaks Italian in the streets. So I learned quickly. I still remember my Italian to some extent. That was also the time of political demonstrations. The communists, the anti-communists. So for example, I still remember the communist song:

FIG. 2. *Murad in Venice, Italy.*

"Avanti popolo, alla riscossa,
Bandiera rossa, Bandiera rossa.
Avanti popolo, alla riscossa,
Bandiera rossa trionferà."

Vladas Pipiras: So your Italian came essentially from the streets.

Murad Taqqu: Yes.... Many years later, I wanted to go back to Italian, so I started reading and my Italian is not that bad. In fact, I had a student at Boston University, Alberto Montanari, who wrote a thesis partly with me in Italian. And then at one point I ended up in Italy (Figure 2), and I gave a one-month course of lectures in Italian. You know, it worked very well, because when you lecture in their language the students tend to be very nice.

Vladas Pipiras: Returning to languages, French was kind of native for you, as you studied it in Lebanon and then in Italy, and in Switzerland growing up. Then, you had German for the gymnasium? What other languages do you speak?

Murad Taqqu: Yes. Hebrew because, you know, we were Jewish, and with family in Iraq and in Israel. Hebrew is a language which I speak quite fluently.

Vladas Pipiras: What language was spoken at home when you were growing up?

Murad Taqqu: Ah, with my mother in general it would be French, and my father spoke mostly Arabic. So, for instance, if he needed to buy a suit in Switzerland, he would go and choose the suit, and then bargain on the price of the suit.

Stilian Stoev: So the bargaining went well in Arabic in Switzerland?

Murad Taqqu: Yes, that's a whole point. I mean, it's inconceivable for someone who can hardly speak anything but Arabic to start bargaining in the first place, let alone bargaining in Arabic in Switzerland, for the price of a suit. But he didn't care. And finally, it always seemed to have worked. He made himself understood. It's a different culture. That's the whole point.

Stilian Stoev: Tell us more about your Arabic. Is it only spoken or can you read and write?

Murad Taqqu: Arabic is made up of dialects but there is also literary Arabic. In school people learn the literary language, and in the street they speak the spoken version of the language, and one person from one dialect cannot easily talk to another person who knows a different dialect. They have to use literary Arabic. Arabic for me was the language that my parents spoke between them or in Iraq, and all that was in the Jewish dialect. In Palestine, it would be the Palestinian (or Levantine) dialect, and in Saudi Arabia, an altogether different one. The most different ones are the dialects in Algeria, Morocco. The Arabic there is very different from the Arabic I am familiar with. So that's Arabic.

Vladas Pipiras: You didn't see Arabic at school, correct? So was everything coming from the family? Did you learn the written Arabic language with them too?

Murad Taqqu: No, so what happened was that I never really went to school in Iraq. I couldn't read it, but could speak the dialect, which I still can to some extent. But again, if somebody knows Arabic well they may not be able to speak a particular dialect, but would be able to understand and speak literary Arabic, which is in a way the *lingua Franca* between all dialects of Arabic.

When I went to Boston University (BU), I said to myself that I'm at the university, so why not take advantage of it? I have sat in courses in Arabic at BU. The courses at BU were literary courses about how to read and write. So I ended up learning those things. These were my only classes in Arabic.

Rachelle now has decided that she wants to learn to speak Palestinian Arabic. So, I won't be able to understand her (laughter).

Vladas Pipiras: Any other languages you speak in addition to English, Italian, French, German, Hebrew, and Arabic?

Murad Taqqu: I learned Russian a little because I ended up in Russia with the Academy of Sciences exchange. I tried to learn as much Russian as I could at the time there, but I forgot it as fast as I learned it, so I can't claim I know Russian: "это очень трудно для меня говорить" (Figure 3).

FIG. 3. *Albert Shiryaev and Murad Taqqu at the Vilnius Probability conference in Lithuania*, 1981.

1.3 Recollections About High School, EPFL and Their Professors

Stilian Stoev: Returning to your studies, you went to study undergraduate math and physics at the *École Polytechnique de l'Université de Lausanne* (EPUL), which is now renamed *École Polytechnique Fédérale de Lausanne* (EPFL) in Switzerland. When and how did you become interested in these subjects, and why both?

Murad Taqqu: Well, when I was in high school I was in science. I mean those who are not doing the classical track, basically end up in science. I ended up in probability, but that's jumping ahead. Let me tell you a story. At the time in high school in the summer, they had projects that you could do, and if you did the project, then you could win a money prize. So I did this and ended up doing a literature project. At the end of the summer, I won the first prize. But the first prize in literature didn't give you a lot of money. So I decided that I would try again the next year. I would look at the list of all proper fields, and choose the one that gives you the largest money prize. It turned out to be mathematics—probability. So, I had a list of problems to solve and this is how I ended up learning probability.

By the way, I won the prize and I was able to buy a camera with the money.

Vladas Pipiras: Did you do the problems on your own, or with somebody who was mentoring you?

Murad Taqqu: No, on my own.

Vladas Pipiras: Interesting. So even in high school, it was already clear to you that this is what you want to get into.

Murad Taqqu: That was the natural route. It was clear that I was going to major in math or science given the situation—because of what I knew, this was the simplest thing to study.

But I also tried learning literature for example. I recall I was in a theatre play by Ionesco. You know mathematics wasn't something that I decided to study at the very beginning. I just fell into it, in a way. I liked it.

Stilian Stoev: Can you tell us a few words about the university system at the time. What was most memorable about it?

Murad Taqqu: It's not very different, really, from the current university system. When I was at the university, I basically had to study the sciences again. It was an applied university *École Polytechnique*—like MIT if you will, although obviously this is not an attempt to compare it with MIT. Basically my concentration was physics, not math, but I had to take a fair number of math courses. If you took some more math courses they would give you a math degree. That wasn't the most important for me, because the most important was the physics part. Nevertheless, I got a degree in mathematics, and eventually it turned out that it is with the help of that degree in mathematics that I ended up at Columbia University.

Vladas Pipiras: Were you a good student?

Murad Taqqu: I was a good student!

Vladas Pipiras: Were the teachers good?

Murad Taqqu: Yes. In fact, one of them was named Georges de Rham. Well, I mean I wasn't really at the level of his research. He was a very well-known person who later played an important role in my life. He was one of my math professors. I took his complex analysis course. It was very good.

And then there was another teacher who was a character, who came all the time dressed in an army uniform. His name was Ernst Stueckelberg. He was in physics (thermodynamics) and he wrote on the blackboard with tiny letters everywhere and then, when the blackboard was filled, he jumped to places where there was a little space, and continued writing in that little space and then jumped again, so he never erased anything.

Vladas Pipiras: What did you like more at the time, math or physics?

Murad Taqqu: Both. I still like both math and physics. I like the physics or even the physics in math, because it extends your horizon. So the math that I was doing, in a way, was closer to physics, I guess.

Stilian Stoev: Other than languages, math, and physics, what other subjects did you enjoy?

Murad Taqqu: Theater. In fact, at some point during my university studies, I ended up in a place where they teach actors. They let me in, even though I didn't have any money. It was not a theatre troupe but a kind of theatre studio—a place for beginning actors. A place to learn and practice acting. If somebody couldn't make it, I would take their place. So it wasn't performing. It was just learning, but this was important for me.

Vladas Pipiras: Did you end up actually, like you said, replacing someone and performing?

Murad Taqqu: Very small pieces. I mean they were negligible roles, but for me it was a big deal.

Stilian Stoev: Do you find that the skills that you learned there helped you become a great presenter and teacher? Or do you think a teaching talent is something altogether different?

Murad Taqqu: I don't know. I always approached undergraduate teaching a bit like performing. Undergraduate, not graduate teaching!

2. GRADUATE SCHOOL IN COLUMBIA

2.1 From Physics to Probability

Vladas Pipiras: When you were in Switzerland, your main professional degree was in physics. The math degree seems an appendix in a way. Why did you switch from physics to math?

Murad Taqqu: When I graduated from the EPFL, I wanted to apply to a physics program. I applied to the Ph.D. program in physics at the University of Maryland at College Park, where I ended up staying only a year. After a few months I was too… *dépaysé* ["out of place", in French]. I was, you know, in a culture shock. It was a university in the South, the University of Maryland, and I came from Europe. It was a totally different culture for me. So I decided to apply to another university.

I had more friends in New York than in Maryland, even though I was living in Maryland. So I went to the Graduate Statistics Department at Columbia and asked them if I could apply. They said, sure you can apply. So I applied to the Graduate Statistics Department at Columbia and ended up there at a time when they had about four, maybe five, professors there. One was Herbert Robbins, another one was David Siegmund. Tze Leung Lai was a fellow graduate student. There were not too many people, but they were all very good.

Vladas Pipiras: How did you end up in Maryland? So when you applied to the physics graduate programs was that the only place you applied to?

Murad Taqqu: Well, I applied to three, four universities, and got accepted by one. So I didn't have much choice, and it was a good university. That was a time, the Cold War, when physics was an important field. There were many students who were getting into physics. But I viewed it as very frustrating, because you study physics and at the end you get a job in industry, not physics, because there are not too many academic jobs in physics.

So Maryland was a place where you had a lot of people finishing and trying to find a job but doing something different from what they were studying. Far from the theories that they were studying.

Vladas Pipiras: How did you decide to go to the US in the first place? You were in Switzerland. Did you think of staying in Europe and applying to graduate schools in Europe rather than coming to the US?

Murad Taqqu: I got to the US as a graduate student. It was much easier. In fact, up to that point I was a stateless refugee. I didn't have any passport or anything like that during all those years. So even coming to the United States, I needed special papers, and all of that.

Vladas Pipiras: Are you saying that this would have been much more difficult in Europe?

Murad Taqqu: Much more difficult. In the US there were many more opportunities to get into the system. In fact, at Columbia, you could have a Master's degree while doing your Ph.D. If you write a Master's thesis, you get a Master's degree. Nobody does this because they're working for their Ph.D. But I wanted to do the Master's, because if you have a Master's degree, you are viewed as a professional, and if you are a professional, then you can get a green card more easily. As a stateless person, I needed something like the green card. So I wrote a Master's thesis. The only place it exists is at the library at Columbia University. I don't exactly remember the topic [66].

Vladas Pipiras: How did this switch from physics to math happen for you? You mentioned that it was difficult for physicists to get jobs in the field. But was there any other factor that affected your decision?

Murad Taqqu: There was a social part also. I ended up having more friends in New York somehow.

Vladas Pipiras: Why did you not switch to physics in Columbia? Why switch to statistics? Was it just much easier to get in?

Murad Taqqu: I think so. You know, I hardly had any credentials.

You are asking a good question, because a lot of those things in my life happened by chance. It was a bifurcation.

Stilian Stoev: Whom did you end up working with at Columbia? I know Benoit Mandelbrot was, in a way your co-advisor. Who was your main advisor at Columbia?

Murad Taqqu: In addition to Mandelbrot, I had another, nominal advisor. Burton Singer, a student of Samuel Karlin. You know, jumping ahead, Karlin played an important role in my life.

Vladas Pipiras: But you also did a Master's at Columbia. Was it with a different adviser?

Murad Taqqu: No. Basically, I had the same adviser for my Master's and Ph.D. at Columbia but he was just a nominal adviser.

Vladas Pipiras: What was your Ph.D. thesis about?

Murad Taqqu: My Ph.D. thesis was on self-similarity, the beginning of self-similarity [67]. Basically taking dependent random variables and trying to do a functional central limit theorem which gets you fractional Brownian motion. And then, if you take the limit of sums of squares of long-range dependent variables, you get the so-called Rosenblatt process. I am the one who named it the Rosenblatt process (Figure 4). So that was the paper on the weak convergence to fractional Brownian motion and to the Rosenblatt process [68].

Vladas Pipiras: Was that at the suggestion of Mandelbrot to look into this?

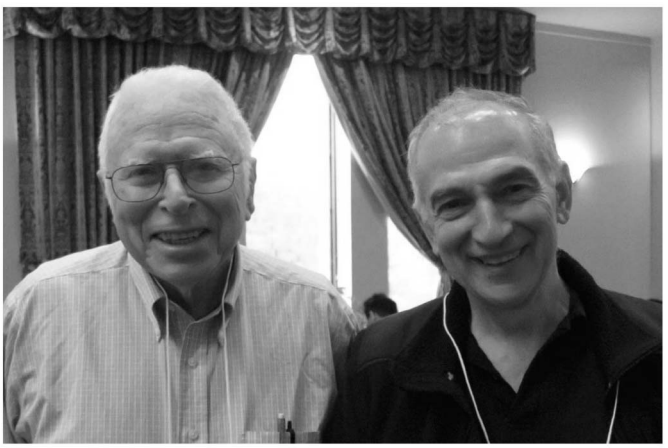

FIG. 4. *Murray Rosenblatt and Murad Taqqu at the conference in his honor, UNC-Chapel Hill*, 2012.

Murad Taqqu: No, that was my idea. I went to IBM as a summer intern essentially, and I learned about fractional Brownian motion. I then asked myself what would be the limit if you considered cumulative sums of squares of long-range dependent Gaussian variables. This is how you get the Rosenblatt process. So I started in that direction, because I was hired in the summer to use FORTRAN, basically to program things. Mandelbrot gave me this programming job which got me interested in the question.

2.2 Benoit Mandelbrot—Personal and Philosophical Influence

Stilian Stoev: Can you tell us more about this programming job at IBM. Were you assigned to Mandelbrot?

Murad Taqqu: The story goes like this. In my first summer at Columbia, I wanted to get a job because, you know, to survive essentially.

I applied to all kinds of positions but I was turned down. I realized that it wasn't a good way to do it, and for the following year, I decided to do it differently. I went to each professor, to each person I knew, and asked that person if they knew of a place which would be interested in getting a student to help. And so, in fact, my advisor at Columbia, Burton Singer, told me about Mandelbrot.

So, I called Mandelbrot and introduced myself. I wanted to ask him about a job, but he ended up, as he usually did, talking about one thing after another. Basically he took over the conversation by himself essentially, and very soon, we switched from English to French, which impressed him, and I mentioned I had taken a course of Georges de Rham. He was very impressed at that point. It was very funny how those totally disjointed things ended up having an influence on my life like this....

After an hour-long discussion on the phone, Mandelbrot said: "I don't have a job." That was it. But then, a few months later, and before that summer still, he called back and said that there may be a place for me at IBM. IBM had a summer program for student interns, and so I was assigned to Mandelbrot, naturally, because he was the one who put my credentials forward.

I was very impressed by Mandelbrot because first of all I understood a little about where he came from. It was sort of more the European way of doing things, and I learned a lot because he spoke a lot, and said very interesting things. So you don't want really to stop him. And he was very helpful to me. In fact, I recommend his book [44], which is very interesting and basically his autobiography.

I was quite overcome, reading this book very recently. When I was with him, he hardly talked about his life, about the war, about what he did during the war, but everything comes together in that book. It's very interesting, and he was somewhat idiosyncratic. But you know that's okay.... You don't have to be like everybody else (Figures 5 and 6).

FIG. 5. *Benoit Mandelbrot and Murad Taqqu during Mandelbrot's 70th birthday celebration in Curaçao in* 1995.

Vladas Pipiras: When you started working with Mandelbrot, was he already a fractal celebrity—the fractal celebrity that we know now?

Murad Taqqu: He was not known as a fractal celebrity at that time yet. Fractals came near the end of his career. It involved scaling in some fashion. The hydrology part was quite at the beginning, and there was also his work

FIG. 6. *Mandelbrot and his wife, Aliette Kagan, Curaçao in* 1995.

on statistical linguistics, Zipf's laws [42]; and the various models that came from it. But he was looking for the big ideas.

Shuyang Bai: Mandelbrot was an unusual mathematician, quite special and very nontraditional, who was more focused on big ideas like scaling. But your thesis research is really a very precise math, right? Proving limit theorems for fractional Brownian motion and the Rosenblatt process. What was Mandelbrot's reaction to your thesis work?

Murad Taqqu: He wasn't very interested in it. Let's put it this way. He was very happy with the results, what came from it, but he didn't follow what I did in my thesis, or afterwards. But I was also very independent.

Vladas Pipiras: Did Mandelbrot have more students around that time, or were you the only one?

Murad Taqqu: At that time, I think I was the only one. There was van Ness, but he was a colleague, somebody working with him. Also, there were a number of people who got involved with describing fractals or painting fractals and the mathematics of fractals. He was very pleased that the topics that he was talking about are so beautiful [7]. For instance, he checked what happens in music, if you have an H in music. What's the sound, is it beautiful or not beautiful [80]?

Shuyang Bai: Can you tell us a bit more about what your interaction with Mandelbrot was like?

Murad Taqqu: I think it was a little bit like a father and child in some sense, but he had a very complicated life, and I didn't know about it at all. The autobiography [44] that I mentioned was also a revelation, because I didn't know most of the things there, all the European part, the war... this type of thing. And it's really through this book that he is revealing this to the public.

So I wasn't really influenced by that part of his life because I didn't know about it. But... he (Mandelbrot) was not a detail person in mathematics but he had good ideas and he had intuition. He was frustrated that people did not respect him for that. Therefore, for him, people were either in his camp or not in his camp. But, for me, Mandelbrot was very much like a father.

3. A LONG-RANGE CAREER PATH

3.1 Israel and Samuel Karlin

Walter Willinger: Did you go to Israel right away after your Ph.D.?

Murad Taqqu: Almost right away, but there is an interesting story with my Ph.D. thesis. My thesis was handwritten. When I passed my defense, the committee said that the thesis was interesting, but they wanted me to revise and type it, and gave me an extension of three months. Rachelle happened to need to do some research in England, so we went there for three months, where I was working on revising the thesis. I had to find a typist. I found one who didn't know any math at all, but it really worked out well.

After the thesis was typed, it had to be delivered to Columbia University. Rachelle had a friend who was going to the United States, and he said that he could deliver the thesis for me. So, the day came. There was traffic and I was late. The plane was about to leave. I argued a little bit to be allowed on board, which they actually agreed to do, together with the stewardess (flight attendant), and we walked on the plane. I kept repeating the name of the person. I had no idea what he looked like but he raised his hand saying, here I am, and he was totally shocked. He could not believe that something like that would happen. That is the story of how my thesis got delivered.

Vladas Pipiras: Could you speak about your years in Israel?

Murad Taqqu: Yeah. So, the reasons to go to Israel were twofold. One was that we are Jewish, and it was an important place. The second one was that Rachelle was doing her dissertation research on Palestinian history. My first job I had there was a visiting position in the Department of Statistics in Hebrew University of Jerusalem. In fact, I taught in Hebrew, so that was a very good practice for me. I don't know if it was good enough for the students, but they were very accommodating.

Walter Willinger: So you stayed in Israel for two years?

Murad Taqqu: Yes, one year I was in Jerusalem, and the second year I had a postdoc position at the Weizmann Institute in Rehovot. The Weizmann Institute felt like a campus of an American university.

Vladas Pipiras: What made you switch from one place to the other?

Murad Taqqu: Because of Samuel Karlin. He gave me the position. Karlin was a professor at Stanford, who then went to Israel to create a new research group at the Weizmann Institute in probability. He offered me the postdoc position.

Not only that, but Karlin was working with Howard Taylor, who was a professor in Operations Research at Cornell. Thanks to this connection, Cornell subsequently

hired me without a formal interview. Karlin was an incredible person, someone totally devoted to research day and night. He has written a number of books in all kinds of directions. Many of you probably know the texts by Karlin and Taylor [34, 35]. So everything was converging to Cornell. In a way, it's a little bit like Mandelbrot's life. My progression was by chance: if I had not met Karlin, I may not have been able to end up at Cornell. But it was a lucky move.

3.2 The Cornell Years and Visit to the Soviet Union

Herold Dehling: How many years did you spend at Cornell?

Murad Taqqu: I got my Ph.D. in 1972, spent two years in Israel, and then stayed at Cornell until 1985, so it's eleven years.

Walter Willinger: What was appealing for you at Cornell? Was the Department of Operations Research a good place for you?

Murad Taqqu: I was in the Department of Operations Research, but the department affiliation was not that important. What was important was a field you associate with, which is something like a virtual department—it's a group that can get together and define something. So you can be part of a seminar and not part of the department. It was a very good and interesting experience.

Vladas Pipiras: So you interacted a lot with people outside your department?

Murad Taqqu: I interacted a lot with the Math Department. The probability theory group was very good: Dynkin, Spitzer, Kesten, and others. They had this regular seminar every week, and I participated in it. It was very interesting. They also invited people to come to the seminar. The differences between the personalities, for instance, of Spitzer on one hand, and Dynkin on the other, were extreme. For example, when a speaker came and gave a talk, Dynkin could say: "You know what is a good talk? If people can understand it, that is a good talk." So, in other words, clarity instead of the details and all that. Spitzer was very bothered by this type of statement. Anyway, there were interesting interactions and that had a good effect. Dynkin at some point created his own seminar, which met once a week, and he spoke on whatever he was working on during that week. He was basically presenting ongoing raw research. It was too technical and you didn't have the big picture. Then he would come the following week and say: "All I did last week was bad, erase it. Don't do that." It was interesting in terms of work in progress, but otherwise it was totally incomprehensible. But then, at some point, when I heard him presenting his work at a conference with all the context, I thought: "Oh, my dear, that is what he was talking about!"

Shuyang Bai: When you were at Cornell, was there an overlap with Kiyosi Itô?

Murad Taqqu: Yes, and I took a course from him—obviously, not as a student, but you could sit in. It was very interesting and he was a very good teacher. I viewed myself as lucky to be able to go to those courses, with Itô and others. I view it as though my undergraduate education took place in graduate school, and I got my graduate education in my professorship. Unfortunately, Itô had to go back to Japan because he had to be with his mother who fell ill. That's why he left Cornell. He later visited Boston at some point. There was a conference at MIT in honor of Itô and Norbert Wiener, both being pivotal in the development of stochastic integrals, and he came back to the United States to attend. I haven't seen him since.

There is also another important thing that happened during my time at Cornell. In 1978, the International Congress of Mathematicians (ICM) took place in a place next to Helsinki, Finland. At that time in the Soviet Union, there was great discrimination against Jews, but nobody outside of the Soviet Union really knew much about it. And basically the way it worked is that, if you were Jewish, you couldn't find a place to present your talk. And it was extreme. Dynkin had suffered through it. He basically ended up in a department of Economics which accepted him. I don't know all the details, but he became a very important name in Economics, for example, for his model of economic equilibrium. But all this happened because he did not have a position in a Mathematics department. So, Dynkin wanted to protest about it. Before I went there, Dynkin, my wife Rachelle, myself, and a number of other people worked out a protest letter. Rachelle, I remember, put it into good English. It was a one-page statement. So we went there and distributed the statement that described what was happening to the Jews. It was very specific with names and dates and exactly what was happening. The next day the entire Soviet delegation at the meeting walked out. So a small group of us from Cornell essentially helped raise awareness about that situation [see also 38].

Stilian Stoev: You mentioned you visited the Soviet Union. Did Helsinki happen after your visit to the Soviet Union?

Murad Taqqu: It happened before my visit to the former Soviet Union. I was invited for a month there by Roland Dobrushin. They had housing for foreign visitors, which belonged to the university or some entity that invited you.

And when I went there, I was sort of wondering whether I was being spied on or not. I remember what I did: I had a few books on a shelf in some specific order, with a page turned a little bit here and there, and I noted that on a sheet of paper for myself to see if someone came and tried to look in my affairs. After I returned, I checked my sheet of paper, and there was no change. So nobody spied on me as far as I know.

I have nice recollections from my visit there. I did give a talk eventually. To give a talk, you had to be sponsored by someone so that you could be added to the seminar list. You needed to have written permission, I believe, to be able to give a talk. And again, Dobrushin invited me. I was speaking about self-similarity and things like that. Yacov Sinai was there, a few other people I knew were there. Because ultimately I didn't know Russian, I spoke in English. Ten minutes into the talk, somebody disagreed and interrupted, and then somebody else counter-interrupted. Then the whole discussion was going on all the way, and in Russian also because they couldn't sustain the conversation in English. I had my camera. So I started taking pictures of people arguing, and no one was interested in me anymore. So that was funny.

Vladas Pipiras: Did anything else stick with you from that visit?

Murad Taqqu: The other event I recall concerns the *refuseniks*[1]—scientists who had lost their jobs— because again there was pressure from, for example, the Steklov Institute. You could not even have a paper published freely. Every paper had to be submitted to a committee in the Institute, which had to approve it.

Because of this situation, the refuseniks, mostly Jews, had to smuggle books or papers outside the Soviet Union because basically everything they did was rejected from publication. You were not allowed to send a paper that you were writing to a journal abroad, for example. While there, I was asked to take a chance and basically be a scientific smuggler. That is, to take a few papers, and when I left the Soviet Union, to submit them in the name of the person who wrote the paper to journals. The problem was that you had to be very careful because it was not allowed. I remember my departure from the Soviet Union. At the airport, I mean, before getting into the plane, you had to pass customs. They would check what you were taking out of the country. So obviously, there was some risk that they would fine me at least and throw away the papers. I went to the airport early, before I was supposed to. I studied the guards there and waited until they were in discussion. They started arguing with each other about something that I did not understand and they did not pay much attention to me. So I waited for the right moment and I went through, and they let me pass without inspection because they wanted to continue their argument. It was quite an experience. Because it was an official meeting, I had special connections with the American Embassy in the Soviet Union. So I had certain privileges, but I was warned not to do something like that by the Embassy. And they gave me a map of Moscow that they asked me to give back before I leave. When I showed this map to Russians living there, they were totally stunned. They had never seen such a detailed map of Moscow. Anyway, I really enjoyed my stay in the former Soviet Union.

[1]Refuseniks were citizens of the Soviet Union, mostly Jews, who were denied permission to leave the Soviet Union. In the sequel, many lost their jobs. If they had been scientists, they could only continue to do research privately in their home.

3.3 Moving to Boston

Vladas Pipiras: Speaking of Rachelle, what was her professional journey? How did you manage that balance together?

Murad Taqqu: What she did when we moved to Ithaca, where Cornell is, she completed her dissertation and then taught history for five years, at Cornell, Stanford, and at a nearby college in upstate New York. But at that time there were very few tenure-track positions in her field anywhere in the country, so eventually she decided to switch fields and go to business school. So, the main reason we left Cornell was because of the opportunities for Rachelle. We needed a big city. Eventually I got a job offer in Philadelphia at the University of Pennsylvania and one in Boston at Boston University. I decided to go to Boston and joined BU.

Herold Dehling: So when you moved to Boston, three of your students or collaborators also moved to the Boston area. Was it by chance or did they go there because they knew that you were moving?

Murad Taqqu: Not totally by chance, I think. But only partly due to the fact that I moved to Boston. But again Robert (Bob) Fox, for instance, was already in the area, before I moved. Florin Avram became aware of the job in Boston. I can't really tell you their motivation, but it did make sense for them to come to Boston too. Also, we had this seminar at Boston University which was of great interest, but obviously they didn't come because of the seminar. Certainly that made it more lively by having different people.

And this seminar was something I started when I went to BU. Basically what happened is when I got my job at BU, one of the conditions was that they would allocate ten thousand dollars a year for seminars that I would organize. There was no seminar in Probability and Statistics

at Boston University at that time. There was one at MIT with Richard Dudley and Molly (Marjorie) Hahn. But I wanted to have something broader, and I wanted people to have the opportunity to learn about the subject to be covered in the seminars because otherwise you just get the new results and you don't really have the perspective, and it's not always easy to understand. We had the seminar every week on Thursday, for two hours in the morning, and then in the afternoon at the regular time that one has seminars—around four o'clock—to have the talk that the speaker was going to give, which is a bit more conventional. This worked in all kinds of interesting ways, because you can expand the horizons, and people were able to get to the root of the matter. You know, no speaker expected that to happen, but they had to go along with it.

In the end it worked out well. And what happened also was at North Carolina, there was Stamatis Cambanis at UNC-Chapel Hill, who, among other people there, Ross Leadbetter and Gopinath Kallianpur, had a big grant from the Army, to organize the Center for Stochastic Processes. So he attracted a number of people to spend a year or so there, and those people were all very good, and also of interest to myself, since the topics often involved limit theorems, dependence and stuff like that. Cambanis unfortunately passed away young.... But the research place in North Carolina, the one he organized, endured, and so did those people. So, there was a source for me to get good people of interest as speakers for my own seminar. These included Liudas Giraitis, Gennady Samorodnitsky, and a few others. Basically, I was able to invite them to come to BU first to give a talk, and then to spend a year and a half as a research fellow. So the existence of this seminar was unusual but very interesting for everybody involved. I feel that it's my contribution of basically attracting people. It also transformed the research atmosphere in the area at BU (Figure 7).

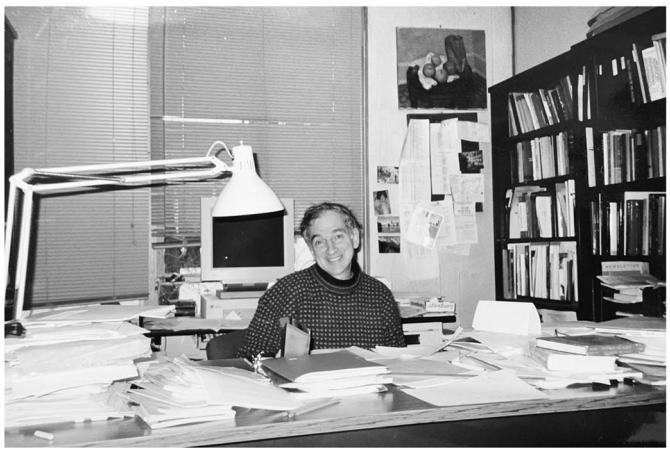

FIG. 7. *Murad in his element*, *Boston University* (*early* 2000*s*).

4. A RESEARCH JOURNEY WITH LONG-RANGE DEPENDENCE, HEAVY TAILS, AND BEYOND

4.1 First Papers: Long-Range Dependence and Noncentral Limit Theorems

Stilian Stoev: Your first paper [65] came out of your summer job at IBM with Mandelbrot, correct? People may be surprised to learn that this paper was published in the *Water Resources* journal. Can you tell us a bit more about it?

Murad Taqqu: Yes. So this is how it happened. At IBM, I started working on that particular problem which was basically, the R/S statistic among the first estimators of the long-range dependence parameter H, introduced by Hurst himself [31].

I wanted to program it in FORTRAN. Somebody else had programmed it, but didn't do it correctly, so that it didn't have a straight line as it should theoretically have a straight line. So that was my first paper [65] (see also [45]). It was, basically, to slightly correct the starting point, which influenced the results a lot, because you did a regression from the beginning. Mandelbrot was very happy that I could show him that, in fact, it works better than he thought it would.

Stilian Stoev: But your Ph.D. thesis ended up being a lot more theoretical. You studied the limit behavior of sums of long-range dependent Gaussian time series and obtained convergence to fractional Brownian motion, right? Did you discover the role of Hermite polynomials in your thesis or did this come later?

Murad Taqqu: It was in my thesis. There can be convergence to fractional Brownian motion but also convergence of the sum of the squares of long-range dependent Gaussian time series to the Rosenblatt process [68]. Rosenblatt had written a paper a long time before and called it *"Independence and dependence"* [61]. He is the one who introduced the notion of strong mixing (or alpha-mixing) [60]. Rosenblatt wanted to show that the process that he had (Gaussian time series with auto-covariance of the form $1/(1+|h|^2)^{2\gamma}$ for time lag h with $0<\gamma<1/4$) was not strong mixing. He took the square and he didn't get convergence to a normal. So that was the reason why he basically introduced this setting.

And when I looked at that, and I said to myself, I don't really see why should you stop at power of two. What if you took the third power? I realized very soon that I could have an expansion and I could use the terms of the expansion. This is how the Hermite polynomials came about.... I did not know at the time what would happen for the higher terms in the polynomial, but I realized that one can use it to discover the hidden structure. Particularly, the difficulty was that if you have something of order two, which

is the Rosenblatt process, the moments characterize the distribution. So you can have a proof by using the method of convergence of moments. If you had terms higher than two, you could have convergence of the cumulants, but... does that show you physically what is going on?

In that early paper [68] I did not discover the role of the higher order Hermite polynomials. I had proofs trying to show the sum goes to a stochastic integral, so that you could see what happens, but it wasn't the best way of doing it.

Walter Willinger: You made a lot of contributions in this area of Noncentral Limit Theorems, whether it's on the probabilistic side, on the modeling side, or on the inference side. Are there particular contributions that you are especially proud of? Which are your favorites, and why?

Murad Taqqu: One of the first papers, the one on convergence to the Rosenblatt process [68], opened a new area. Before, we were all having Brownian motion and fractional Brownian motion in the limit, and not going out of the Gaussian situation. The paper is really reinterpreting a result of Rosenblatt [61], and this is why I called the limit the *Rosenblatt process.* So it's really asking the question, first of all, can it be true for processes instead of just marginal distributions, which was what Rosenblatt did. But the second thing is, what about other powers, other nonlinear functions. What happens? And then we get this reduction principle [68–70], which says, if you expand your random variables in Hermite polynomials, it's only the first nonzero term that matters. That opened a new door, really. What are the conditions for this to happen? What are the limits of Hermite polynomials of Gaussian time series? That opened a big door. First were the Hermite polynomials, then afterwards, what if you take just linear transformations of non-Gaussian white noise. So next, instead of having functions of Gaussian processes and asking what happens if you go to the limit, you consider linear transformations and the kernel of these linear transformations gets you long-range dependence? But now we don't even have functions of Gaussian processes. In the limit you are going to get whatever you get, maybe the same thing as if it were Gaussian, but to show it required work... and that opened another door.

Shuyang Bai: Is there an extension of the Hermite polynomial expansion approach to the non-Gaussian setting?

Murad Taqqu: This is a natural question. Instead of taking Hermite polynomials, what do you take in that situation? This got me to the Appell polynomials, which the Hermite polynomials are just a particular case of. So you get a much bigger picture.... This was done with Florin Avram [4].

A number of papers were in that direction [26–30, 78] but, in fact, the problem with Appell polynomials was that in general, for the Gaussian case, any L^2-function of the Gaussian process, can be expressed in terms of Hermite polynomials. But for the Appell polynomials, you can only deal with polynomials, so it's not satisfactory in the same sense as it is for the Gaussian case, and this is why I think, there were a number of papers in this direction, but it is not a complete story because of that. But it still has a certain beauty.

Shuyang Bai: Did you meet Rosenblatt when you were doing your thesis research?

Murad Taqqu: I met Rosenblatt when he came to IBM for a week or so. We talked about it and... he found it interesting, but I also think he was very pleased that I called it the *Rosenblatt process*. In fact, I felt that it was really only fair to call it this because he introduced all of that.

Herold Dehling: One of the most influential papers that you wrote is the paper of the convergence of integrated processes of arbitrary Hermite rank. There is a similar approach to the same kind of results by Dobrushin and Major. Can you comment on that?

I recall that both your paper [70] and Dobrushin and Major's paper [17] were published back to back in the same issue of the Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete.

Murad Taqqu: It was two things: there was the paper by Dobrushin and Major [17] and later Major also wrote a book on iterated integrals [41]. The book was very good.

With regard to the papers, it was this paper by me [70], and the paper by Dobrushin and Major [17]. The paper by Dobrushin and Major was much better than mine as far as I am concerned because I remember one argument in my paper—I was trying to prove convergence in distribution by using the convergence of moments. The convergence of moments doesn't directly imply that you have convergence in distribution, so I had a cumbersome argument. They had a much nicer argument, but basically we came at the same time, more or less. So these two papers were very important, but Dobrushin and Major's paper is better than my paper, as far as I'm concerned.

Herold Dehling: Had you been in interaction before with Dobrushin and Major, or was this completely by chance that you all worked on the same problem?

Murad Taqqu: No, our contacts came after the papers and, in fact, I met Major and Dobrushin in Esztergom, which is a town in Hungary. Dobrushin invited me afterwards to come to the Soviet Union (Figures 8 and 9).

FIG. 8. *Left to right*: *Dobrushin*, *Murad Taqqu*, *and Yaglom in Vilnius*, 1981.

It was through him. He was the head of the Institute for Telecommunication. But at the time in the former Soviet Union, people went to a department or a school, where a job existed. So, for example, even Dynkin, he couldn't stay where he was. He found a job at the Institute of Economics. I don't know the name of the institution.

So, Dobrushin became the head of telecommunication.... He was able to invite me, to make the arrangements, so that I could stay in the Soviet Union for a month or so.

Walter Willinger: Are there pieces of your work that you thought deserve more attention? That you thought—Oh, look, this is exciting—but then you know the response wasn't as as much as you expected. Are there such things?

Murad Taqqu: There are things. Yes, but it was all in the direction of long-range dependence and all that. For example, the law of iterated logarithm [69]. There is an interesting article on the functional Law of Iterated Logarithm with Claudia Czado [72]. She was a Ph.D. student at Cornell. I suggested that area or the problem to her. And then I left Cornell, and she continued working with somebody else, and I think she is working in Munich, Germany, now.

Herold Dehling: Yes, eventually she went back to Germany, and she is a professor of Statistics at the Technical University of Munich.

FIG. 9. *Murad Taqqu attending a probability conference in Vilnius*, 1981.

Murad Taqqu: Right. As far as this long range dependence is concerned, the problem was very interesting, I felt, because when you have the law of iterated logarithm the statement is formulated in terms of lim sup and lim inf or something like that and if you do it functionally, you get a ball, and approaching that ball in a function space.... It's very elegant when you can see it through a functional law of iterated logarithm, but since I guess I wasn't there anymore, it was a good thing for Claudia to change topics. So the paper [72], I think, is very interesting, but not well known at all.

4.2 Statistics of Long-Range Dependence

Shuyang Bai: Could you comment on the statistical aspects of long-range dependence? In fact, your first paper was statistical [65] on the estimation of the Hurst parameter H. Then came your path-breaking contributions on the probability side regarding noncentral limit theorems and convergence to fractional Brownian motion, the Rosenblatt, other self-similar processes of higher Hermite rank [68–70]. What about the statistical side of the story?

Murad Taqqu: There is the probability side, and then, again, you have this Hurst parameter H, or $d = H - 1/2$, or whatever, which characterizes dependence. And how do you use statistics to estimate it? That's another question.

I think the important early paper in this direction is the paper with Fox [23], which has been quoted very often. It is on how to use the Whittle estimator to deal with the situation where your data is long-range dependent. The interesting thing about this situation is the story behind that very important paper. Originally what happened was, we had a long paper, which dealt with defining the estimator, and we got its limit, in the context of the Whittle estimator. So the Whittle estimator is the statistical estimator that allows you to estimate the parameter d or H, in a situation where it is difficult to work with the likelihood function in the time domain. And using the Whittle estimator really worked very nicely in practice. But that paper was long, and had essentially two parts. It had the introduction of the statistic and the discussion of the statistic, and then what was needed, limit theorems, to get you what was necessary. I felt that the paper was not going to be publishable, because maybe, people interested in one side would not care about the other side. So basically, I suggested having two papers, one a probability paper, one a statistical paper. And that made it very acceptable by the statisticians, because it worked very well practically, and for probabilists, that had a limit theorem that was of interest by itself. So, if you wanted extensions, you can go now in one direction or the other direction with their own lives, and that really, made the whole thing work in practice, you know. So I think the other probability paper appeared in *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete* [24]. By the way, it was this paper and a few other papers on similar types of topics that were also published in that journal. But at some point, well, also they changed the name of the journal (now called Probability Theory and Related Fields). That's a small thing, but I haven't had a single paper published by them since a certain time. It was very funny, I mean, even the papers written together with somebody else. I think the reason is that it was the direction of the journal, which really changed totally. It became much more physics related.

Walter Willinger: So which one ended up being sort of the most successful, on the probabilistic side or the statistics one.

Murad Taqqu: The statistics. Yes, because the statistics paper had all those applications, and the probability had just a probability result that can be extended and so on but it wasn't really as popular. But overall I would say that there were many follow-up papers that expanded this area in all kinds of directions.

Walter Willinger: Indeed, you were involved in many related developments yourself, on various long-range dependence estimators in different settings [1, 74], as well as in their applications to real data [51, 64, 75, 83]. Do you feel there is still work left to do in this direction?

Murad Taqqu: Yes. For instance, with respect to the comparison of wavelets with Fourier. Also, multidimensional data. The book *Long-Range Dependence and Self-Similarity* [55] contains bibliographic notes that list a wide variety of extensions (see also the collection of papers [18]).

4.3 Heavy Tails

Vladas Pipiras: Long-range dependence was one major running theme in your research work but then you also made fundamental contributions concerning heavy-tailed and stable distributions. How did you get involved in this direction? Was it under Mandelbrot's influence again for these distributions represent another form of scaling?

Murad Taqqu: Yes. I think it was Mandelbrot's influence.

Shuyang Bai: Can you tell us more about the influential book on stable processes [63] you wrote with Gennady Samorodnitsky? How did this whole project start? How long did it take?

Murad Taqqu: It started first with the seminar that took place at UNC-Chapel Hill with Stamatis Cambanis. Gennady got a fellowship there. The year after, he came to

BU, and we ended up being interested in stable processes. It was very frustrating that there was nothing really systematically available for stable processes, besides the classical approach. So we decided to write something, and it expanded, expanded. It took about four or five years to have the book written, because there were many results in small areas. And it was clear that it was necessary to try to put them together. And so the reason, partly, why it took so long is that when you work on something, there is a natural question that you would like to solve, to be able to continue working, and you work on that question, and this expands the field. So I thought that it would be important at this stage to have some material like this, for somebody else coming, to be able to use this thing. And I also wanted to emphasize clarity because I really like, if possible, to have everything be clear and well explained, more than just having the result.

And so the book was written. In fact, I have a very funny story about it. Because when the book was almost ready, you would be getting the galleys. In other words, a copy just before being printed. I insisted on seeing two or three pages of those galleys, because they are very expensive. They don't really use them besides for verification. I insisted on choosing a few pages, and looking at them. There was a certain amount of resistance by the publisher, but nevertheless they did it. When the thing arrived, by mail, I was horrified. It was illegible, because all the letters were strange and distorted. The formulas as displayed didn't mean anything. It was like a mixed-up picture. I went back to the publisher, who changed it. But it would have been a catastrophe if this hadn't been changed. So, this is what happened with that book. But then the book, by the way, became very popular. You know, there are still a few copies being sold every year afterward, I don't know, for thirty years or so. I think what we did was to make it very clear. And I think that, in general, this is the advice I would give to anybody who writes a book at this point is: emphasize clarity. Put in details. Because you want people to understand what's going on, instead of having to rework out the details, because the authors assumed they were obvious, or whatever. So this book is still a classic in some sense.

Stilian Stoev: I still remember the first time I encountered your book with Gennady Samorodnitsky [63] when I was an undergraduate student in Bulgaria. I was shocked. It was a very sophisticated mathematical book, which you could open on any page, and maybe you won't understand some details or the context, but you could still read and follow. I was really influenced by this book.

Piotr Kokoszka: Do you remember that I became your Ph.D. student because of this book?

Murad Taqqu: Yes, I sent the first three chapters of the book to your advisor in Poland, Aleksander Weron, with a request for comments. He asked you to read them and a few months later I got your comments. I am happy you ended up coming to BU to do your Ph.D. with me. You proofread and offered suggestions for several other chapters, and your thesis included topics related to the book [36, 37].

4.4 Generation of Long-Range Dependence and Telecommunications

Walter Willinger: Can you talk about your work on the generation of long-range dependence through the superposition of renewal reward processes? You were one of the pioneers of this formulation.

Murad Taqqu: So that's also originally due to Mandelbrot [43], as far as I know. Again, it wasn't very precise with all the details and all of that. But he had this idea. In fact, the key motivation of Mandelbrot was basically trying to put together dependence and superimposing it together. So he already had the idea that you get convergence like that. Because he was interested in scaling. In the whole work of Mandelbrot, really, the main theme is scaling. So anything that ultimately gets you scaling in the limit, is something that he would be interested in. Scaling also involved infinite variance. So, he was trying to put those things together. I think that he is the one who basically introduced and developed that methodology from a theoretical perspective.

In fact, my book with Eberlein [19], contains the following paper with Joshua Levy: "Using renewal processes to generate long-range dependence and high variability" [73]. Using alternating renewal processes, we showed convergence. It wasn't very complete in terms of proofs. Then it said that the details of the proofs will appear somewhere else. Eventually, they did appear [in 57].

Walter Willinger: Can you tell us more about the book you just mentioned? I think that was your first book. I remember you worked on this for quite a while and a couple of people were involved.

Murad Taqqu: So let me say a few words on that book [19]. It's called "Dependence in Probability and Statistics. A Survey of Recent Results." There was a meeting at Oberwolfach[2] of the various people that were involved with this topic. And, I was basically behind the push that it should be a book with papers that give overviews of things, instead of very specific theoretical interests. There are, in this book, really papers that are still interesting now, because they give you a general perspective and general results. For example, Herold Dehling wrote there on

[2]Oberwolfach Research Institute for Mathematics (MFO).

almost sure approximation of U-statistics. I am there with Levy, using renewal processes to generate long-range dependence and high variability, Walter Philip is there on invariance principles for independent and weakly dependent random variables. Manfred Denker is there. Anyway, you had all kinds of very interesting general articles. For instance, the paper by Bradley "Basic properties of strong mixing conditions", as well as Magda Peligrad's article, which were used a lot. If you have the book, you should keep it!

Herold Dehling: I will keep it (laughter). It has been extremely influential, and the articles in the book have been cited extensively. Basically, this was an excellent idea. You edited this book with mostly survey papers, and there, you could really review the state of the art in the field. The authors represented the list of who is who of those days on dependence in probability and statistics.

Murad Taqqu: So one thing I remember also is that while everybody was there in Oberwolfach, I basically talked to each person separately, saying,"You do this, you do that, and you do this…", instead of just letting the person do whatever he or she wanted to do.

Water Willinger: In a sense, you outsourced writing a book (laughter)!

Murad Taqqu: Yes, but it was very interesting (laughter). Going back to the generation of long-range dependence through renewal processes, let me also say that the application of this to telecommunications is due to you, Walter.

Walter Willinger: Well, the application, yes, but it was basic….

Murad Taqqu: But you put it together!

Walter Willinger: But I remember, when you said that it goes back to Mandelbrot, he tried to explain why in the stock market you encounter long-range dependence, didn't he?

Murad Taqqu: Yes, I don't know if it's the stock market coming from the renewal processes. It may be coming more from your side. I think the situation was that I was at Stanford for sabbatical, and you (Walter) were at Stanford at the same time, searching for what you wanted to do. We were together and talking about all kinds of things. And then I convinced you to come to Cornell.

Walter Willinger: You were able to tell me that Cornell is a better place than Stanford (laughter).

Murad Taqqu: Yes, something like that (laughter). Anyway, so I was very glad that you accepted to come to Cornell. And for thesis, what we tried to do was relevant to the stock market. The emphasis in this area was on martingales and their applications in finance.

Walter Willinger: I mean, you let me work in that area. You were always interested in the stock market and related things, I mean, for many, many years, no?

Murad Taqqu: Yes, but you worked on the stock market. Then you got your Ph.D., and then you went to work for Bellcore. After a while, a reasonably short while, you realized that it (stock market analysis) would lead you nowhere in the world of telecommunications. One needed something relevant to telecommunications. So basically you had the Ethernet data and trying to model it, and you said, what about long-range dependence in this situation? You were the key person to make the connection between the two things who could have made the connection. And also, I remember, I taught a course on long-range dependence at Cornell and you were there. So you remembered that model that you had been aware of. I got involved because of you. Also, you had the language and the technical knowledge that have to do with how the Internet traffic measurements are done. And the methodology made sense in telecommunications, because of realizing that you had long-tailed file sizes like the heavy-tailed on/off processes. So basically it was a very good thing that happened.

Walter Willinger: I got self-similar processes and long-range dependence for four years, which came in very handy, because I knew about the basics of the estimation techniques that you described. I knew about this Mandelbrot construction that you were working on, too. So they all came together very nicely. But I think the important thing in terms of your work on this generative method, demonstrating that you have long-range dependence, or scaling phenomena, that don't necessarily have anything to do with critical phenomena or phase transitions, which was sort of at the early stage assumed as given. That didn't have any appeal for engineers. But your construction had an enormous appeal from an engineering perspective, and so that construction, really, I think, opened up a whole new way of mathematical modeling that people did not have before. In that sense, the work you started with these generative methods of long-range dependence, in terms of modeling, has had an enormous impact [10, 40, 77, 81, 82].

4.5 Empirical Processes

Herold Dehling: We did this work on the empirical process of long-range dependent data, and actually discovered some very unusual phenomena there, that the limit

process is completely different from what one would have expected. Do you remember how we actually got into this? I recall we originally wanted to work on U-statistics of long-range dependent data, and this somehow led us to the empirical process.

Murad Taqqu: Yes, the U-statistic was interesting. I didn't know about it. But then it was the obvious question: The U-statistic, asymptotically, can involve multiple integrals and then long-range dependent data can also have representations through multiple integrals. And so what happens when you put those two different ways of getting multiple integrals together? And this was the beginning of the questions we were asking.

Herold Dehling: Yes, this was the motivation to look at the U-statistic of long-range dependent data. But then this somehow led us to the empirical process. Of course, it's a very fundamental object in nonparametric statistics, and I recall that we even had a seminar on Pollard's book that had just come out then on the general approach to empirical processes. I'm not sure any more of whether we did this on purpose to prepare for the work when the data are long-range dependent, or whether it was just helpful.

Murad Taqqu: Yes, I don't recall myself, but I think it was again one thing leading to another.

Herold Dehling: Yes, it could be like that, indeed. There was a lot of empirical process theory, of course, in the area. There was Dudley at MIT, who had been immensely influential, on that subject. And so for me, part of the motivation was that I just wanted to know about what they were always talking about.

Murad Taqqu: I remember you did excursions walking in the forest, the hills and the mountains, with Dudley, you being a fellow climber.

Herold Dehling: Yes, that was indeed, my second association with Dick Dudley. Dick was actually the editor in chief of the Appalachian Mountain Club White Mountain Guide. So he would regularly organize hikes in the White Mountains, and he would always have a book with him to write down whenever he saw any kind of changes in the trails. He would also write down the time that it would take you to get from one place to the other. Dick also had a cottage up in the White Mountains.

Stilian Stoev: I think people are yet to rediscover some of your research, especially in statistics. The techniques that you have developed on dealing with long-range dependence are going to be quite useful more generally. For example, I am aware of a study in the field of shape restricted inference (see, e.g., [3, 5]) where your early work [68] and your paper with Herold Dehling on empirical processes were instrumental [16].

4.6 The Fourth Moment Theorem and Diagram Formulae

Stilian Stoev: Recently you published a book in collaboration with Peccati [54], which deals with moments of multiple integrals, diagram formulae and the like. There is also that fourth moment theorem. How do you look at the developments in this area?

Murad Taqqu: The diagram formula is a way to present the moments of functions of Gaussians. This is one thing I considered because I was using the method of moments. But the important thing is ultimately the convergence in distribution. Nualart and Peccatti [52] showed that in the context of Wiener chaos of fixed order, the convergence of the first four moments is enough for convergence to the normal distribution. Even though the convergence of moments is not enough to get convergence in distribution in general, they showed that basically in this setup the four moments are enough. The first moment is zero. The second moment is variance, so the convergence of the third and fourth moments to zero and 3 times the squared limiting variance yields asymptotic normality in the setting of multiple integrals. And that was big. This is the fourth moment theorem.

And in a way, if you think about it, it's not that astonishing that the situation like this holds because you are making a huge assumption. For example, if you have convergence of the second moment and assume independence, you're going to get a convergence to normal. That's the central limit theorem. Adding independence is a big deal. You are basically killing all the other moments and just getting the normal. So it's not that astonishing to be able to find a situation beyond independence, where the convergence of additional moments gives you convergence to normal. The question is what is this additional stuff that you are doing, and in this case, it is looking at multiple integrals. And then in the book with Peccati, we tried to do it with the convergence of multiple Poisson integrals and other types of situations, more or less of the same type [54].

Stilian Stoev: Indeed, the Wiener chaos is a very rigid structure. Is there a counterpart of the fourth moment theorem, if the limit is the Rosenblatt distribution?

Murad Taqqu: Well, there are many questions like that. There is a paper by Bourguin, Campese, Leonenko and myself [12], where the random variables are sort of diffusion type. But in this whole area, I think, the results depend very much on the assumptions you make on your random variables. Yes, this is quite an open area still.

5. REFLECTIONS AND PERSPECTIVES

5.1 Mentors and Influences

Shuyang Bai: You mentioned that you effectively worked independently, except chatting here and there with Mandelbrot. You also mentioned the influences of people like Karlin and Rosenblatt. Did you have any other mentors, I mean, people who helped you? Were there any other researchers that you looked up to?

Murad Taqqu: I was more or less independent. I think there were other people who indirectly helped me, because their papers, or the works they did, were useful. For example, when I was in Israel, the first year I was there at the Hebrew University, Henry McKean was there too, and he had just published an article in the Annals of Probability, which was just being created. First, there was the Annals of Mathematical Statistics journal, and then they divided it into the Annals of Statistics and the Annals of Probability journals because there were too many papers, and they needed a new series. I also remember that because one article in the first issue of this new journal (Annals of Probability) had to do with Hermite polynomials and in fact perhaps the first exposition on Wiener chaos by McKean [46], and I approached him, and we talked about it. He explained to me what he was doing, and it was very interesting. In many ways, I guess, a lot of papers, including your papers (e.g., [6]), Shuyang, also involve Hermite polynomials.

5.2 Teaching and Mentorship

Stilian Stoev: You have been a very successful mentor. What qualities should a beginning graduate student have or develop to be successful in probability and statistics research?

Murad Taqqu: Well, that's true for other things too, not specifically for probability and statistics. But I would say it is to be enthusiastic about the project or whatever idea that you have, and pursue it to some extent, and drop it if it doesn't lead you anywhere. In practice, you would stay with it. Try to figure it out. And it's, in fact, not really that difficult in some sense. You can view the whole thing of science as a big ball. And you are looking at a very small area of that big ball. And so you start with it. You have some background because there are some papers about that subject. And it's enthusiasm that you need really. You shouldn't view it as a duty that you have to do. But it's rather a mystery. It's like a detective story, trying to find who did it, or something like that. And be enthusiastic. And then things go much simpler, much smoother this way. Because if it doesn't work, you just leave it on the side. You don't even have to necessarily drop it. But at least you would have thought already about things like that. And I think in most cases, this is really how it works. You also have an advisor. In other words, it's a question of the choice of advisor. But that's another story. The advisor helps you to place yourself in that big ball and presumably knows a little bit of the area. It's a way for you to get into it and make it work if it works.

Stilian Stoev: You were also immensely productive. Some people probably find it hard to start a problem, but they have less of a trouble finishing it. For others, it is the other way around. Is there any secret behind your productivity? How do you complete a paper?

Murad Taqqu: Well, the completeness of the paper is somewhat arbitrary. The question is whether there is going to be an obvious sequel. Or is that it? But I think you should definitely have time to explore your idea a little bit. Because if you don't explore it, you're not going to develop it. You're not going to have it. You won't know if it's useful, if you can use it later, or if it's effective. So it's worth spending the time to get into it to finally decide whether one can then pursue it further or not. But how long do you have to be able to reach that stage? That depends on the problem. And the key thing is really the enthusiasm, the excitement. That's the motor. That's the engine. It pushes you. And that's key, because if you don't have it, then nothing works. I mean you can't have other people enthusiastic about your ideas if you yourself are not.

Stilian Stoev: That's right. I think this might be a key to teaching too. Students remembered best the parts of the lecture where the lecturer was most enthusiastic, rather than anything else.

Murad Taqqu: Yes, it's like a show, you know, it's theater. So you remember certain parts which are particularly exciting, even though they may be secondary in the show.

But I think really even a research talk should have a part which is exciting, in the sense that it's like a mystery you want to solve. So you start by indicating the background. And then finally, you are getting to the mystery. And so you are making the audience aware of the mysteries like: Oh, what's going to happen now? Who is going to get killed, so that it creates a certain amount of excitement.

Stilian Stoev: You've been notorious among your students and collaborators to sit and work with them for three or six hours at a stretch, editing papers, working out through technical arguments. Is this one important ingredient to your success? What can you say about your modus operandi?

Murad Taqqu: Having the details somewhere is very important, because you want to make sure that what you

are saying works, and you have to work it out, to check whether it works. And so particularly when I write a paper, I like to put in more details rather than fewer, because I want to make it easy for the reader to read it, because that attracts the reader also. Besides that, you have to check that what you are doing is correct. So statements like "it is obvious that", or something of this particular type, is not very useful, because if it's obvious for you, it may not be as obvious for somebody else. You shouldn't be afraid to go and work out the details and present them.

Walter Willinger: You spent lots of time with students, refining, revising papers. Looking back, how did you find all that time? I remember times where we would work from eight to midnight, or something like that. And, you know, the next morning you go and teach, and then you have your kids, everything together. It's really amazing to see how you made time for this.

Murad Taqqu: I don't know. For me working with good students, motivated students is a pleasure. So by spending more time than usual, makes it even more interesting. So it's not an effort. You know, you're having ice cream and ice cream and ice cream.... For example, with Walter, we stumbled on this nonstandard calculus thing in finance. We had a discrete version of the Black-Scholes model [76]. Area experts (Nigel Cutland, Ekkehard Kopp in the UK) looked at it and that led directly to an application of nonstandard analysis in this area [14, 15].

Walter Willinger: This was in the thesis that I wrote with you, trying to do the pathwise stochastic integration. In the discrete world, it turned out to be very nice because you could relate it to primal-dual problems in linear programming, and that was a very nice formulation. And then the continuous case was this huge pain. But you let me work on it and somehow we were able to finish it.

Murad Taqqu: Your contribution is really quite extraordinary because you did one thing, you did another thing, you did the third thing. And to some extent, you even found a relationship between those various things at a high level.

Stilian Stoev: I remember a career advice you gave me, when I had to decide what job to take after graduation. You said: *Pick a place where you will be able to continue to learn*.

Murad Taqqu: Right. In fact, there is a joke that I heard from somebody from Israel. He was telling me about two people walking in the forest. Both of them were professors. So they were talking, and one of them said I got a job offer somewhere else, and I don't know exactly what to do, and I have these things telling me not to do it, and that was a situation telling me to do it. And so the other professor says, since your field is operations research, why don't you just optimize. Look at the various pieces, put them together, and just get the best. And the first person said: Oh, I can't do that! This is serious!

Walter Willinger: Your enthusiasm for learning was contagious and unique. I remember that you would sit in courses, lectures and take notes like a student. It was something you were interested, you wanted to hear, and very often, I think other people wouldn't even realize that there was a professor sitting there...

Murad Taqqu: By the way, it's really funny that my view of myself is, in many ways, different from your view of me. I mean, what I did, for me it was something natural. I couldn't see it as something special in some sense. So the way of teaching, the way of mentoring, it's all connection with people. It's really fun. For instance, I am not the type of person who would work in his corner and not talk to anybody for months. So it's the attraction of being with people, intelligent people and having fun through this.... So it's a very natural thing to do, without in fact thinking about it, because that's how it was. So it's very interesting for me to hear what you are saying now. It was natural, and if I didn't have that, I don't think I would have been interested in research or anything like that. It was the human connection (Figures 10–14).

Herold Dehling: It may be very natural for you, but your mentorship, thinking and actions had great impact on students, often changing the trajectory of their lives. To take me as an example, I was a postdoc for two years at BU. For me, it was very much a turning point in my life. It was very difficult in those days getting permanent jobs anywhere in Germany. And, so, I was extremely happy to have this post-doc position, which was actually a visiting

FIG. 10. *Murad Taqqu with Hira Koul and Liudas Giratis at the conference in his honor, UNC-Chapel Hill*, 2012.

FIG. 11. *A group picture at the conference in honor of Murad Taqqu*, *UNC-Chapel Hill*, 2012.

FIG. 12. *An informal group picture at the conference in honor of Murad Taqqu*, *UNC-Chapel Hill*, 2012.

FIG. 13. *Murad Taqqu with students and colleagues at Boston University.*

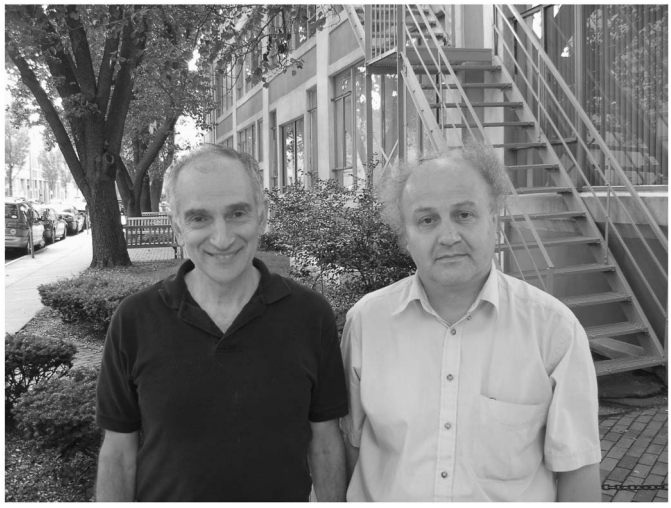

FIG. 14. *Murad Taqqu and his collaborator Nikolai Leonenko outside the old building of the BU Mathematics and Statistics Department.*

professorship. I got new research directions through our collaboration, Murad, which then eventually secured me my first permanent job. And without this time in Boston, I wouldn't have stayed in academia. So that is for sure. And I felt that you were indeed always extremely supportive, and I also remember these long hours we sat working on papers and discussing on the blackboard.

Murad Taqqu: Yes. I may have influenced a few students... Jan Kallsen comes to mind. He was a student of Ernst Eberlein who came to BU from Germany. He was unsure what direction to pursue and ended up working with me on mathematical finance. We wrote an interesting paper in the area [33]. He went back to Germany and has been a professor in Germany. He is very well known in the field.

Herold Dehling: Yes, we are both examples of being invigorated by BU experience.

Stilian Stoev: You have been unconditionally supportive and generous with your time and mentorship to students, colleagues, regardless of their backgrounds, such as women or minorities.

Murad Taqqu: I was taking the people as they are, whether they are women or not, or minority or not. You choose the person based on enthusiasm. So you had very good students who could apply and get into a university in the United States at a graduate school. That was a way of having students like Renata Cioczek-Georges, Norma Terrin, Yingchun Zhou and others.... I can understand that a high administrator would view it as a statistic: how many people here and how many people there. But it doesn't really work that way. Because ultimately you are talking about an individual.

5.3 Family and Work

Vladas Pipiras: You and Rachelle married right around the time you graduated from Columbia. Can we ask how the two of you met?

Murad Taqqu: Another story. That was in New York. I had a woman friend who was getting married, a big Jewish wedding. She felt that there were too many men but not enough women. So she called my apartment in New York, trying to reach the woman I was subletting from, a mutual friend, but of course I was the one who picked up the phone. She was a little embarrassed because at one point she had invited me, quite casually, and she felt that she had to invite me. I went to that wedding and met Rachelle there. She was a graduate student at Columbia. But nothing happened for a few months.

Then we saw each other again. At the time, tea was served in the graduate lounge on weekdays at four o'clock– right after the period when graduate colloquia were scheduled. This was an old-fashioned ritual, where faculty wives poured tea and served cookies. And so this is how I found Rachelle again.

Vladas Pipiras: Did Rachelle get her Ph.D. in history from Columbia?

Murad Taqqu: Yes. She taught history for several years but there were few tenure-track academic positions available in her area.

Stilian Stoev: I wanted to ask a few other questions about your personal life. You once told me that you volunteered in your children's kindergarten. You had a story about one of their teachers being amazed. How can you teach graduate courses in probability? And then a few minutes after that, switch to playing with children, or vice versa? How do you remember that time? More broadly, how did you manage to balance family life, teaching, and work?

Murad Taqqu: That was in Ithaca. Our daughter was enrolled in a cooperative nursery school where the parents had to volunteer. When it was my turn, I had to leave quickly to go teach a class at Cornell. The nursery school teacher was astonished that I could shift so fast from nursery school to college. I told her it was more or less the same thing. Only the subject was a little different.

But, generally speaking, I loved Cornell because, it was a very good place, and, in fact, very flexible in terms of the departments not being too important, obviously important in terms of teaching and all that, but in terms of interests, you could associate yourself with many groups.

It was more difficult for Rachelle. She has a Ph.D. in history but was teaching as an adjunct professor, nothing

permanent. It was not a very good way to develop a career. Just before our sabbatical at Stanford, she decided to go into finance. And so at Stanford she taught history to upperclass students and took some freshman calculus classes to prepare for business school. She got her MBA at Cornell. And as I mentioned earlier, that's how we eventually ended up in Boston.

Herold Dehling: I know that you're very proud of your children and your grandchildren. Could you to tell us a bit about what they are doing?

Murad Taqqu: Yael, our oldest, studied at Yale University and later got an MBA at Harvard. Her undergraduate major was Neuropsychology. She was also interested in music and piano. And the Yale School of Music is excellent, but it's a graduate program that admitted undergraduates for music lessons on audition. Yael managed to do a lot of piano work and performance there, and she also became the student conductor of the Yale Glee Club, and even made a tour of Asia together with the Glee Club. After college, she joined the consulting firm of McKinsey, where she's now a partner. She's married with three teenaged children.

Jonathan, our youngest, has been running a data-focused tech startup that helps patients get better access to medication. Jonathan went to college at Brown University and majored in Applied Math and Economics. He's quite multilingual. He got an MBA and a Master's of International Affairs at Wharton. He has two very young children (Figure 15).

5.4 On Research Areas

Stilian Stoev: You have done research in both probability and statistics. You have seen the two fields evolve and coexist. How do you view their interaction over the years?

FIG. 15. *Murad Taqqu and his family. UNC Chapel Hill*, 2012.

Murad Taqqu: I think it's an important question. There are two fields or two domains of research, probability and statistics, and depending on the problems, you choose one or the other. Really, if one thinks about it, you have on one side, the extreme side, you have pure mathematics. So in some sense probability can be close to the pure mathematics. And statistics can go into very specific applications. But in probability, you also have applied probability. So I think that for somebody working in probability, you have the luxury of working on something, and then shifting to work on something else. So it's like giving you lots of opportunities to do interesting things in a very wide area. You're not going to do everything, but you can go in one direction versus another direction, and have sub-areas in some sense almost created by your own research. It's a luxury. I don't know enough of mathematics in general to know whether that's the case in other areas in mathematics–say, algebra, topology, or whatever. But in probability and statistics, you have this incredible flexibility of being on the applied side, on the theoretical side, emphasizing statistics, emphasizing probability. And, in fact, the existence of this connection is what really pushed me to get involved in those areas because I didn't have any area of specific concentration.

Walter Willinger: What do you think of the future research in the areas that you have been working in?

Murad Taqqu: I think it's hard to say. But I think if one can put distinct areas together, supported by models and data, it could become interesting. You can then expand it in all kinds of directions. And in fact, this is what happens if you really look at what we all worked on, historically. The questions that you posed came in because of the notions that people had on the problem. So in other words, you have A and you have B, you put A and B together. And A is an area, and B is another area. And how can you put A and B together? And is it going to be interesting, this A and B together? Because very often, in fact, it's very elegant. I think of heavy tails, stable distribution, and all that. You have all those connections of areas waiting to be explored.

Shuyang Bai: Murad, thank you very much for responding to our questions and for sharing your thoughts. We very much enjoyed the interview.

Murad Taqqu: Since my bibliographical guide on self-similar processes and long-range dependence [71] a lot has changed. While this is not an attempt to provide an up-to-date review of the literature, I wanted to end with a *list of books* that have been published in the areas of *long-range dependence*, *heavy-tails* and *self-similarity*. I felt that it is important to list at least some of them that come to my mind.

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- Beran (1994) *Statistics for Long-Memory Processes*, Chapman & Hall/CRC [8].
- Samorodnitsky and Taqqu (1994) *Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance*, CRC Press [63].
- Edited by Adler, Feldman, and Taqqu (1998) *A Practical Guide to Heavy Tails: Statistical Techniques and Applications*, Birkhäuser [2].
- Embrechts and Maejima (2002) *Selfsimilar Processes*, Princeton University Press [21].
- Edited by Robinson (2003) *Time Series with Long Memory*, Oxford University Press [59].
- Embrechts, Klüppelberg, and Mikosch (2003) *Modelling Extremal Events: for Insurance and Finance*, Springer Science & Business Media [20].
- Edited by Doukhan, Oppenheim, and Taqqu (2003) *Theory and Applications of Long-range Dependence*, Birkhäuser [18].
- Palma (2007) *Long-Memory Time Series: Theory and Methods*, Wiley-Interscience [53]
- Resnick (2007) *Heavy-Tail Phenomena: Probabilistic and Statistical Modeling*, Springer Science & Business Media [58].
- Biagini, Hu, Øksendal, and Zhang (2008) *Stochastic Calculus for Fractional Brownian Motion and Applications*, Springer [11].
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