Supplementary material for the paper "Quantifying the risk of heat waves using extreme value theory and spatio–temporal functional data"

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In Section 1 we discuss relevant previous research. Possible extensions to other extreme weather events are discussed in Section 2. Some additional results for Assessment of Methodology are provided in Section 3.

1 Previous research on modeling heat waves

There is no single definition of a heat wave (Gershunov et al., 2009), though it is generally defined as some period of unusually high ambient temperature. Numerous countries issue heat wave alerts based on such a definition. For example, in the United States, the National Weather Service generally issues a heat warning when, "... the maximum heat index temperature is expected to be 105° F or higher for at least 2 days and night time air temperatures will not drop below 75° F", though this varies somewhat by region (National Weather Service, 2016). Gosling et al. (2008) provide definitions of a heat wave used by specific cities. Another definition requires the 3-day running mean temperature to exceed the 97th percentile for at least three days (Hajat et al., 2002) or the local maximum temperature to exceed the 90th percentile of local summertime temperature during 3 successive days (Beniston, 2004). Abaurrea et al. (2007) define a heat wave as a set of continuous days, of no minimum length, during which the daily maximum temperature is above some given threshold. Huth et al. (2000) use a general definition, with a heat wave being identified as "the longest continuous period (i) during which the maximum daily temperature reached at least T_1 in at least three days, (ii) whose mean maximum daily temperature was at least T_2, and (iii) during which the maximum daily temperature did not drop below T_2," where T_1 and T_2 are specified upper and lower temperature thresholds, respectively. Other works use similar definitions, though the minimum number of days required for an event to be extreme varies: Stefanon et al. (2012) required a minimum length of four consecutive days, while Fischer and Schar (2010) required a minimum length of six days.

Most definitions of a heat wave involve excessive heat levels, frequently defined by some given threshold temperature above which temperatures are deemed "dangerously hot". These thresholds are typically defined in terms of historic temperature data for a given region. Meehl and Tebaldi (2004) and Peng et al. (2011) define the upper threshold T_1 as the 97.5th percentile and the lower threshold T_2 as the 81st percentile of daily temperature maximums. Fischer and Schar (2010) define this temperature threshold in terms of the 90th percentile of historic daily maximum temperatures, defined for each calendar day of the summer season. Abaurrea et al. (2007) use a similar percentile method in selecting a temperature threshold, but use the 95th percentile instead. Stefanon et al. (2012) used a more complex definition for their temperature threshold, using the 95th percentile of a probability density function calculated for each calendar day based on historic temperature data for that day as well as the ten days preceding and following the day in question. Another common method for selecting thresholds is by using specific temperature values used by the local government of a study area to identify "dangerous" heat levels, which varies from place to place based on local climate. For example, Huth et al. (2000) use set values of 30 degrees Celsius (86 degrees Fahrenheit) for their T_1 threshold and 25 degrees Celsius (77 degrees Fahrenheit) for their T_2 threshold, as these temperatures were commonly used for their case study region in the Czech Republic.

Heat wave occurrences lend themselves naturally to being modeled as realizations of a random process. Katsoulis and Hatzianastassiou (2005) model heat waves in Greece using a combination of deterministic and stochastic components. The number of event occurrences, the time of the occurrence, and the magnitude of exceedances over threshold are modeled, respectively, using Poisson, exponential, and Gaussian distributed variables. Abaurrea et al. (2007) model the emergence of heat waves using a non-homogeneous Poisson point process, while also using regression models to describe their intensity. Variables measured for the modeling process include the duration of the heat wave, the maximum intensity (defined as excess temperature over the given threshold), and mean intensity during the heat event. Furrer et al. (2010) use a Poisson point process to model the frequency of heat waves, a geometric distribution for their length, and a conditional generalized Pareto distribution for their intensity. Winter and Tawn (2016) model temperature using a first-order Markov process and heat waves using the Generalized Pareto distribution. Their approach accounts for the possibility that heat wave duration and intensity are linked. Bortot and Gaetan (2014) use a hierarchical approach to model heat waves. The first level of the hierarchy is a generalized Pareto distribution to model exceedances over a threshold, while the second-stage is a latent Gamma random variable to induce temporal dependence in the model. Reich et al. (2014) also use a hierarchical model allowing for both temporal dependence and the underlying generalized Pareto distribution for marginal temperature to change over time, employing Bayesian inference to estimate the likelihood of observing a heat wave. Smith et al. (1997) combine an exceedance-over-threshold method with a time-dependent component via Markov chains. Shaby et al. (2016) utilize a Bayesian hierarchical model with latent state variables indicating whether a day was part of a heat wave event, with a separate two-stage Markov chain used to model the frequency and duration of heat waves. Stefanon et al. (2012) create local probability functions for the observed maximum temperature of each calendar day and tied observed events to spatial grid points, using agglomerative hierarchical clustering to identify heat wave cluster locations.

2 Extensions to other extreme weather events

The approach advocated in the paper in not restricted to the quantification and computation of the probabilities of heat waves. It can be applied to other extreme weather events that are characterized by temporal duration and spatial extent. For illustration, we describe modifications needed for quantifying the probability of a drought. Since precipitation data are noisy, additional smoothing steps may need to be incorporated, as described below.

A drought is characterized by unusually low precipitation over a region for an extended period of time. The duration parameter ℓ could thus correspond to 2-3 months. Suppose we have observed a time series of raw precipitation measurements (mm) at m locations. Denote the raw precipitation measurements by

$$X^R(\mathbf{s}_i, j), \quad i = 1, \cdots, m \text{ and } j = 1, \cdots, T.$$

The data are partitioned into annual curves

$$X_n^{(R)}(\mathbf{s}_i, \cdot) = (X_n(\mathbf{s}_i, t_k)), \quad n = 1, 2, \ldots, N \text{ and } k = 1, 2, \ldots, 365, \tag{2.1}$$

with $t_k \in [0, 1]$. The sampled observations are very noisy and heavy–tailed. To remove the heavy tails, it is common to apply the base 10 log transformation to the raw measurements to obtain

$$Y_n^{(L)}(\mathbf{s}_i, t_k) = \log_{10}(Y_n^{(R)}(\mathbf{s}_i, t_k) + 1).$$

The curves $Y_n^{(L)}(\mathbf{s}_i, \cdot)$ will be very noisy and should be smoothed using a convenient technique. Denoting by $Y_n^{(S)}(\mathbf{s}_i, t_j)$ the values of the smoothed curves of $Y_n^{(L)}(\mathbf{s}_i, \cdot)$, we construct the standardized annual curves $Z_n^{(R)}(\mathbf{s}_i, \cdot)$ by setting

$$Z_n^{(R)}(\mathbf{s}_i, t_k) := (Y_n^{(R)} - \overline{X}(\mathbf{s}_i, t_k))/\mathrm{SD}(\mathbf{s}_i, t_k),$$

where

$$\overline{X}(\mathbf{s}_i, t_k) = \frac{1}{N} \sum_{n=1}^{N} X_n^{(S)}(\mathbf{s}_i, t_k) \text{ and } \mathrm{SD}^2(\mathbf{s}_i, t_j) = \frac{1}{N-1} \sum_{n=1}^{N} (X_n^{(S)}(\mathbf{s}_i, t_k) - \overline{X}(\mathbf{s}_i, t_k))^2.$$

Note that we suggest computing $\overline{X}$ and SD using the smoothed curves, as they reflect typical long term behavior. However, the $Y_n^{(L)}(\mathbf{s}_i, t_j)$ are retained in the standardization process in order to retain the structure of the extremes in the observed data. The exact definition of the $Z_n(\mathbf{s}_i, t_j)$ would have to be determined after a detailed exploratory data analysis.

To compute the probability of a drought, we argue as follows. A drought occurs in a region if Z_n is sufficiently small for a relatively long duration over an adequately large spatial domain. Consequently, for the duration functional $\mathcal{D}_\ell$, one might choose $\mathcal{D}_\ell^{\mathrm{avg}}$ or

$$\mathcal{D}_\ell^{\max}(Y)(\mathbf{s}_i, t_k) := \max_{l=k-\ell+1,\ldots,k} Y(\mathbf{s}_i, t_l).$$

Similarly, for the spatial extent function $\mathcal{S}_d$ one might choose S_d^{avg} or

$$\mathcal{S}_d^{\max}(Y)(\mathbf{s}_i, t_k) := \max_{\mathbf{s} \in \mathcal{N}_d(\mathbf{s}_i)} Y(\mathbf{s}, t_k),$$

for some appropriately chosen neighborhood $\mathcal{N}_d(\mathbf{s}_i)$. Thus, we might quantify a drought using the functional $\mathcal{H} = \mathcal{S}_d^{\text{avg}} \circ \mathcal{D}_\ell^{\text{avg}}$ or $\mathcal{H} = \mathcal{S}_d^{\max} \circ \mathcal{D}_\ell^{\max}$ or some alternative combination of appropriate duration and spatial extent functionals. Fix an appropriate functional for quantifying drought, and denote this $\mathcal{H}^{\text{dry}}$. A drought will occur if $\mathcal{H}^{\text{dry}}$ is below some threshold of interest.

The probability of a drought in any given year is then

$$p^\dagger(\mathbf{s}_i; \mathcal{H}^{\text{dry}}, u, \ell, d, [a, b]) := \mathbb{P}\big(\mathcal{H}^{\text{dry}}(Z)(\mathbf{s}_i, t_k) \leq u \text{ for some } t_k \in [a, b]\big) = \mathbb{P}(m(Z) \leq u),$$

where

$$m(Z) := \min_{t_k \in [a,b]} \mathcal{H}^{\text{dry}}(Z)(\mathbf{s}_i, t_k) \equiv - \max_{t_k \in [a,b]} [-\mathcal{H}^{\text{dry}}(Z)(\mathbf{s}_i, t_k)].$$

Thus, the GEV estimation can be implemented as explained in the main part of the paper.

3 Assessment of methodology supporting materials

Complete results for one of the simulation experiments used to assess the methodology are provided in Table 1.

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	location	scale	shape	$\hat{p}$
	0.92	0.25	-0.36	0.0000
	1.01	0.18	-0.36	0.0000
	1.01	0.25	-0.58	0.0000
	1.01	0.25	-0.36	0.0000
$\mathcal{S}_d^{\min} \circ \mathcal{D}_\ell^{\text{avg}}$	0.92	0.18	-0.58	0.0000
	1.11	0.25	-0.36	0.0005
	1.01	0.32	-0.36	0.0061
	1.01	0.25	-0.13	0.0212
	1.11	0.32	-0.13	0.0895
	1.11	0.19	-0.26	0.0000
	1.11	0.26	-0.45	0.0000
	1.00	0.19	-0.45	0.0000
	1.00	0.26	-0.26	0.0000
$\mathcal{S}_d^{\text{avg}} \circ \mathcal{D}_\ell^{\text{avg}}$	1.11	0.26	-0.26	0.0002
	1.21	0.26	-0.26	0.0025
	1.11	0.33	-0.26	0.0093
	1.11	0.26	-0.06	0.0205
	1.21	0.33	-0.06	0.0714
	0.08	0.24	-0.32	0.0000
	-0.02	0.17	-0.32	0.0000
	0.08	0.17	-0.01	0.0000
	-0.02	0.24	-0.01	0.0006
$\mathcal{S}_d^{\min} \circ \mathcal{D}_\ell^{\min}$	0.08	0.24	-0.01	0.0009
	0.19	0.24	-0.01	0.0013
	0.08	0.32	-0.01	0.0047
	0.08	0.24	0.30	0.0243
	0.19	0.32	0.30	0.0480
	1.11	0.22	-0.26	0.0000
	1.19	0.16	-0.26	0.0000
	1.19	0.22	-0.49	0.0000
	1.11	0.16	-0.49	0.0000
$\mathcal{S}_d^{\text{med}} \circ \mathcal{D}_\ell^{\text{med}}$	1.19	0.22	-0.26	0.0000
	1.28	0.22	-0.26	0.0008
	1.19	0.28	-0.26	0.0055
	1.19	0.22	-0.04	0.0200
	1.28	0.28	-0.04	0.0682

Table 1: Table of the estimated heat wave probabilities for the Wyoming site, using various combinations of parameter estimates and bounds from 95% confidence intervals. Results are shown for $\mathcal{S}_d^{\min} \circ \mathcal{D}_\ell^{\text{avg}}$ with $u = 1.75$, $\mathcal{S}_d^{\text{avg}} \circ \mathcal{D}_\ell^{\text{avg}}$ with $u = 2$, for $\mathcal{S}_d^{\min} \circ \mathcal{D}_\ell^{\min}$ with $u = 1.75$, and for $\mathcal{S}_d^{\text{med}} \circ \mathcal{D}_\ell^{\text{med}}$ with $u = 2$.

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